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A dynamic evacuation model for pedestrian–vehicle mixed-flow networks

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ABSTRACT

In urban emergency evacuation, a potentially large number of evacuees may depend either on transit or other modes, or need to walk a long distance, to access their passenger cars. In the process of approaching the designated pick-up points or parking areas for evacuation, the massive number of pedestrians may cause tremendous burden to vehicles in the roadway network. Responsible agencies often need to contend with congestion incurred by massive vehicles emanating from parking garages, evacuation buses generated from bus stops, and the conflicts between evacuees and vehicles at intersections. Hence, an effective plan for such evacuation needs to concurrently address both the multi-modal traffic route assignment and the optimization of network signal controls for mixed traffic flows. This paper presents an integrated model to produce the optimal distribution of vehicle and pedestrian flows, and the responsive network signal plan for massive mixed pedestrianvehicle flows within the evacuation zone. The proposed model features its effectiveness in accounting for multiple types of evacuation vehicles, the interdependent relations between pedestrian and vehicle flows via some conversion locations, and the inevitable conflicts between intersection turning vehicle and pedestrian flows. An illustrating example concerning an evacuation around the M&T stadium area has been presented, and the results indicate the promising properties of our proposed model, especially on reflecting the complex interactions between vehicle and pedestrian flows and the favorable use of high-occupancy vehicles for evacuation operations.

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1. Introduction

Mitigating traffic congestion during emergency evacuation has been evolved as one major task for responsible agencies over the past decades. In congested metropolitan areas, commuters are likely to depend either on transit or other modes for their daily commutes. Thus, during an emergency evacuation evacuees often need to move over some distance to their designated locations, such as parking areas or designated pick-up locations, and the massive number of pedestrians may consequently incur tremendous burden to vehicles in the roadway network (see Fig. 1). The difficulty in modeling urban mixed flow evacuation lies in the complex interrelations between these two types of flows, where the vehicle flows generated from parking garages or transit pick-up stations are dependent of arriving evacuees. The conflicts between these two types of flows also need to be effectively addressed by either law enforcement personnel at intersections or with effective signal control strategies. Besides, the vehicle flows typically consist of both passenger cars and transit vehicles, making the modeling of maximizing the evacuee throughput a complex network optimization issue where transit vehicles shall be preferred.

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Despite the increasing number of studies on either vehicle or pedestrian evacuation, the complex issues associated with urban evacuation, such as coordinating vehicle and pedestrian flows, have not been adequately addressed. Hence, this paper introduces a planning model to coordinate both the pedestrian and vehicle flows in an evacuation network. The contribution of our model lies in yielding the answers to the critical issues concerned by agencies responsible for evacuations such as "do we need to dispatch buses for emergency evacuation or not", "how many buses would be sufficient", "what kind of control should we have to coordinate the intersections when both massive vehicles and pedestrians are present".

The paper is organized as follows: next section reviews the existing traffic flow optimization studies for emergency evacuation. Section 3 presents the formulations of the integrated mixed flow network and its key components. Section 4 details an integer-linear optimization model that accounts for vehicle and pedestrian flows as well as their routing strategies within the evacuation zones. Section 5 demonstrates the model application with an illustrative example of the M&T stadium after the football game. Section 6 summarizes conclusions and future research directions as well as discusses the potential applicability of this model in real-world evacuation scenarios.

2. Literature review

The evacuation modeling has received increasing attention since the Three Mile Island Nuclear Incident in 1979. Due to the vast number of studies that has been carried out, we will only review those related to the network flow optimization in this study. The literature review is divided into three categories: vehicle evacuation, pedestrian evacuation, and mixed-flow evacuation.

2.1. Vehicle evacuation studies

Based on the application focus, the vehicle flow optimization literature for emergency evacuation can be divided into the following categories: demand modeling (Wilmot and Meduri, 2005; Lindell et al., 2005; Fu and Wilmot, 2006; Hasan et al., 2011), staged evacuation (Mitchell and Radwan, 2006; Sbayti and Mahmassani, 2006; Chien and Korikanthimath, 2007; Chen and Zhan, 2006), route choice (Hamacher and Tjandra, 2002; Cova and Johnson, 2003; Chiu and Mirchandani, 2008; Zografos and Androutsopoulos, 2008; Ng and Waller, 2010), contra flow (Theodoulou and Wolshon, 2004; Wolshon et al., 2005; Tuydes and Ziliaskopoulos, 2006; Dixit et al., 2008; Xie and Turnquist, 2011), etc. Most early programs, such as NETVAC (Sheffi et al., 1982) and MASSVAC (Hobeika and Jamei, 1985), were based on the "trial-and-error" method, relying on either the macro- or micro-simulation tools for performance evaluation. To optimize the vehicle flow distribution during the evacuation, Dunn and Newton (1992) and Campos et al. (2000) proposed the use of static network flow models. Hamacher and Tjandra (2002) gave an overview of the mathematical models for various evacuation related issues, focusing especially on modeling dynamic network flows, such as maximum dynamic flows, earliest arrival flows, quickest paths and flows, and continuous dynamic flows. On this regard, Ziliaskopoulos (2000) proposed a model with linear formulations, based on the cell transmission concept to produce the system-optimal dynamic traffic assignment to a single destination. Cova and Johnson



Fig. 1. A graphic illustration of Hamburg street@MD295 near the M&T stadium.

(2003) presented a network flow model for identifying optimal lane-based evacuation routing plans in a complex road network. The model is an extension of the minimum cost flow problem. Sbayti and Mahmassani (2006) proposed a set of system-optimal dynamic formulations to schedule evacuation trips between a set of origins and safety destinations. The method of successive average (MSA) was used to find the flow assignment, and a traffic simulator, DYNASMART-P, was employed to propagate vehicles and determine the state of the system. Liu et al. (2006a,b) proposed a cell-based network model to determine the optimal starting time and routes for evacuees in different zones. Yuan and Wang (2007) simultaneous optimized the destination and route choices by solving a traffic assignment problem on a modified network structure. Afshar and Haghani (2008) devised a heuristic optimization procedure to provide a system-optimal solution to the time-varying traffic assignment problem. His algorithm allows for joint optimal choice of destinations, routes and departure times. For more comprehensive review on the evacuation transportation modeling, readers can refer to Murray-Tuite and Wolshon (2013).

2.2. Pedestrian evacuation studies

The second category of evacuation studies focused on optimizing the pedestrian evacuation process, mainly deal with the inside building evacuation scenarios. An early static transshipment network model of building 101 has been widely explored as a benchmark for assessing the applicability of network flow optimization models for in-building evacuation (Francis, 1981). Chalmet et al. (1982) expanded it with dynamic formulations which use the procedure of Ford and Fulkerson (1962) to concurrently maximize the total number of people evacuating from the building for all time periods and also minimizes the duration for the last evacuee to exist the building. An application of EVACNET (Francis, 1984; Kisko and Francis, 1985) was designed later to generate the optimal evacuation plans and to estimate evacuation times for the in-building evacuees.

Choi et al. (1988) modeled the building evacuation by solving the minimal cost dynamic network flows with side constraints of variable link capacities. He proposed the 'greedy' algorithms for some special networks, and developed the solution procedures to take advantage of the unique network structures. Hoppe and Tardos (2000) proposed polynomial time algorithms for the maximum dynamic flow and quickest flow problems with a fixed number of sources and sinks. Lu et al. (2003, 2005) developed new heuristic approaches to find a sub-optimal evacuation plan to reduce the computation cost. However, his proposed heuristics requires the capacity of a link to be constant and independent of the traffic volume in the link. Pursals and Garzón (2009) improved Francis' model (Francis, 1981) by incorporating the evacuation routes and movement equations in the model formulations (Nelson and Mowrer, 2002). Guo and Huang (2012) proposed a continuous space simulation model in replicating the pedestrian movements in the buildings with internal obstacles under evacuation situation.

2.3. Mixed-flow evacuation studies

However, on the subject of simulating mixed pedestrian-vehicle flows over a congested network, only a limited number of studies have been reported in the literature. Milazzo et al. (1998) describes the interactions between pedestrians and turning vehicles using a conflict zone-occupancy approach for more accurate saturation flow rate estimation. Leden (2002) worked on exploring a unique database of pedestrian-vehicle accidents and examined the pedestrians' risks associating with the flows. Zhang et al. (2004) introduced the concept of "stop point" to deal with traffic obstacles and resolve conflicts. His model is intended for efficient simulation of pedestrian's crossing dynamics. Tian et al. (2001) discussed various forms of split-phasing schemes resulting from various pedestrian timing treatments in coordination with the vehicle flows. Yang et al. (2006) presents a pedestrian model for traffic system micro-simulation in China. A time gap distribution extracted from videotape were used to adjust the parameters to match the simulation results with the field results. Helbing et al. (2005) proposed a macro model to investigate the oscillations and delays of pedestrian and vehicle flows. Jiang and Wu (2006, 2007) explored a simple lattice gas model to study the vehicle and pedestrian flows in a narrow channel. Both models presume that the moving directions of both types of traffic flows are not subject to change. Ishaque and Noland (2007) studied the pedestrian traffic with VISSIM, where vehicle and pedestrian modes are operated independently and controlled by traffic signals at the potential conflicting areas. This function has later been expanded in VISSIM to model the conflicts between pedestrian and vehicle flows when any of them need to cross a street using the gap acceptance model (Boenisch and Kretz, 2009). Zhang and Chang (2012) introduced a multi-modal evacuation system for Baltimore city. The system can generate optimal evacuation plans and provide evaluations to the effectiveness of the proposed plan. As a part of the proposed system, Zhang and Chang (2013a,b) presented integrated optimization models to design routing and signal plans for massive mixed pedestrian-vehicle flows within the evacuation zone without including the details of the mixed-flow simulation model adopted in the system. Fang et al. (2013) also selected a stadium and adjacent road network in Wuhan, China as the evacuation environment in experimenting his proposed space-time efficiency model to assess the effectiveness of an evacuation plan.

3. Mixed flow network representation

3.1. Components of the mixed network

The proposed mixed-flow network consists of three main components: the vehicle network, the pedestrian network, and their connections. This study adopts the common unidirectional node-link concept for the vehicle network, and uses the

bi-direction link-node notion for the pedestrian network. However, to ensure the safety and reduce conflicts, the proposed model enforces only one direction of pedestrian flow on each link. To reflect the interactions and conflicts between evacuees and vehicles, the proposed model generates the connection node and intersection nodes to capture such mixed-flow movements. The connection between the two networks is used to convert the pedestrian flows to the vehicle flows, usually taking place at the parking areas and pick-up locations (e.g., transit stop, metro stations). The conflicts between these vehicle and pedestrian flows usually occur at the intersections or crossing areas, where proper signal controls are needed to coordinate these two types of flows.

3.2. Representation of the Vehicle Network

Consider a directed graph $G^{V} = (V^{V}, E^{V})$, where $V^{v} = \{1, ..., n_{v}\}$ is the set of nodes, and $E^{v} = \{(i, j) | i, j \in V^{v}\}$ is the set of directed links. These nodes represent the intersections, and the links denote one-way street that connects two intersections. Fig. 2(a) is an example of two adjacent intersections, and Fig. 2(b) gives the graphic illustration for such connections with the network presentation.

3.3. Representation of the pedestrian network

In general, all pedestrian movements may take place in one of the following areas: inside-building area, sidewalks, and intersection crossings. Since the focus is on guiding and controlling pedestrian–vehicle flows, this study mainly presents our modeling efforts on studying the pedestrian flows along sidewalks and at intersections. Similarly, sidewalks and crossings can also be represented with nodes and links. Different from the vehicle network, pedestrians can move toward both directions on each link, so the pedestrian network is bidirectional in nature. Consider a bi-directed graph $G^p = (V^p, E^p)$, where $V^p = \{1, \ldots, n_p\}$ is the set of nodes, and $E^p = \{(i, j) | i, j \in V^p\}$ is the set of undirected links. These links represent the sidewalks or the crosswalks, and the nodes denote the connections between the sidewalks and the crosswalks. An illustrative example is given in Fig. 2(c) and (d). The solid lines in Fig. 2(d) represent the sidewalks, and the dashed lines denote the crosswalks.



Fig. 2. Graphic illustrations of the network components.

3.4. Modeling the connections between vehicles and pedestrians

During the evacuation process, evacuees are likely to run to the parking garage or bus stops (Fig. 2(e)) where passenger cars or buses are loaded onto the vehicle network to the final safe destinations. The parking garages and bus stops function as the connections between the pedestrian and vehicle networks. There are two parameters associated with the connection: the carpooling rate and the access delay. The carpooling rate determines the ratio between the pedestrian inflows and the vehicle outflows from the connection nodes, and the access delay represents the average time needed for evacuees to access their vehicles which include both passenger cars and transit vehicles. To realistically capture the conversion of the two flows, this study first defines two nodes for each connection site: one in the pedestrian network and the other in the vehicle network. Connection links are then created to connect the set of pedestrian nodes to vehicle nodes in order to transfer the pedestrian flows to vehicle flows. The traverse time in a connection link is equal to its access delay. In Fig. 2 (f), the dotted line, connecting the two connection nodes (shaded), represents the connection link. Note the connection nodes and links represent the mechanisms to reflect the vehicle flow's dependency on the actual pedestrian flow arrivals at parking areas or transit pick-up points, the interaction and conflict between these two flows are coordinated at intersections by means of signal controllers or traffic enforcement personnel, which will be discussed in Section 3.5.

3.5. Representation of the signal controllers

Conflicts may occur between vehicles, between pedestrians, or between vehicle and pedestrians. It is not desirable in evacuations since it may bring about chaos and render the situation uncontrollable. In practice, one can use a traffic control device to enforce the sequence and time for each right-of-way movement at an intersection. A ring-and-barrier diagram is selected to represent the phase sequence of a signal controller. Taking a typical four-leg intersection for example, the conventional signal phase sequence is depicted in Fig. 3(a).

Note that under the typical design, during phases 2, 4 6 and 8, the right-turn vehicles should always yield to the pedestrian traffic. However, during the congested mixed-flow situation such as during evacuation the pedestrian-crossing flow is always over-saturated, which may incur the blockage to the right-turn traffic. To address this critical issue, we split each through-right-pedestrian phase into one pedestrian only and one mainly for vehicles (see Fig. 3(b)). Aside from that, the yellow and all red intervals are combined as one separate phase for the model to include the lost time in the optimization process.

4. Optimal network-wide mixed flow formulation

4.1. Model variables

The decision variables for the mixed-flow network optimization include the dynamic flow distributions for both the vehicles and evacuees in the network, the connection of different movements at intersections, and the signal parameters such as cycle length and green splits. Note all the times are treated as discrete in our model. The notations for those decision variables are listed below:



(a) a typical four-leg intersection



(b) with an additional exclusive right turn phase

Fig. 3. Ring-and-barrier diagrams.

 $\begin{aligned} & \int_{ij,kt}^{u} = \text{vehicle flow from link } (ij) \text{ to } (j,k) \text{ at time } t \\ & \int_{ij,kt}^{p} = \text{pedestrian flow from link } (ij) \text{ to } (j,k) \text{ at time } t \\ & d_{ij,k,t}^{p} = \text{vehicle flow available at node } j \text{ on link } (ij) \text{ to } (j,k) \text{ at time } t \\ & d_{ij,k,t}^{p} = \text{pedestrian flow available at node } j \text{ on link } (ij) \text{ to } (j,k) \text{ at time } t \\ & d_{ij,k,t}^{p} = \text{pedestrian flow available at node } j \text{ on link } (ij) \text{ to } (j,k) \text{ at time } t \\ & d_{ij,k,t}^{p} = \text{pedestrian flow available at node } j \text{ on link } (ij) \text{ to } (j,k) \text{ at time } t \\ & d_{ij,k,t}^{p} = \text{pedestrian flow available at node } j \text{ on link } (ij) \text{ to } (j,k) \text{ at time } t \\ & d_{ij,k,t}^{p} = \text{the fixed interval for the } nth \text{ phase at intersection } i \\ & s_{i,m,n} = \text{the start time of the } nth \text{ phase of the } nth \text{ cycle at intersection } i \\ & e_{i,m,n} = \text{the end time of the } mth \text{ phase of the } nth \text{ cycle at intersection } i \\ & e_{i,m,n} = \text{the end time of the } mth \text{ phase of the } nth \text{ cycle at intersection } i \\ & e_{i,m,n} = \text{the end time of the } mth \text{ phase of the } nth \text{ cycle at intersection } i \\ & e_{i,m,n} = \left\{ \begin{array}{c} 0, \text{ if vehicle movement from link } (i,j) \text{ to } (j,k) \text{ is green at time } t \\ 1, \text{ if vehicle movement from node } i \text{ to } j \text{ is green at time } t \\ 1, \text{ if pedestrian movement from node } i \text{ to } j \text{ is green at time } t \\ & b_{i,j}^{p} = \left\{ \begin{array}{c} 0, \text{ if pedestrian movement from node } i \text{ to } j \text{ is green at time } t \\ 1, \text{ if pedestrians are allowed to walk from pedestrian node } i \text{ to } j \\ v_{i,l,t} = \left\{ \begin{array}{c} 0, \text{ if signal barrier l of signal at node } i \text{ is active at time } t \\ 1, \text{ if signal phase m of signal at node } i \text{ is active at time } t \\ 1, \text{ if signal phase m of signal at node } i \text{ is active at time } t \\ u_{i,m,n,t} = \left\{ \begin{array}{c} 0, \text{ if } t \text{ is earlier than } s_{i,m,n} \\ 1, \text{ if } t \text{ is equal or later than } e_{i,m,n} \\ 1, \text{ if } t \text{ is equal or later than } e_{i,m,n} \\ 1, \text{ if } t \text{$

Variables that are known to the users prior to executing the proposed model include the topology of the network, the capacity and the saturation flow of each link, the locations of the origins and destinations, and the parking lots and pickup locations. The notations of those known variables are listed below:

T: The total time of interest

 $\Gamma^{\nu-}(i)$: The set of vehicle nodes directed to a node *i*

 $\Gamma^{\nu+}(i)$: The set of vehicle nodes directed from a node *i*

 $\Gamma^{p-}(i)$: The set of pedestrian nodes directed to a node *i*

 $\Gamma^{p+}(i)$: The set of pedestrian nodes directed from a node *i*

 S_{ii}^{ν} : The saturation flow rate of the vehicle link (i, j)

 $V^{\dot{s}}$: The set of intersections with signal controllers

V^{*c*}: The set of connection nodes

 E^{v} : The set of vehicle links in the vehicle network

 E^p : The set of pedestrian links in the pedestrian network

E^{*c*}: The set of connection links

O: The set of origin nodes in the pedestrian network

D: The set of destination nodes in the vehicle network

 m_i : The number of evacuees at the pedestrian origin node i

 λ_i : The carpooling rate for the connection node *i*

 U_i^p : The maximum pedestrian holding capacity of the connection node *i*

 C_i : The set of phases of the signal plan at node *i*

 $C_{i,j}$: The set of allowed movements in phase j of the signal plan at node i

4.2. Constraints for vehicle flows in the vehicle network

$$\sum_{k \in \Gamma^{\nu+}(j)} q_{i,j,k,t}^{\nu} = \sum_{k \in \Gamma^{\nu+}(j)} q_{i,j,k,t-1}^{\nu} + \sum_{k \in \Gamma^{\nu-}(i)} f_{k,i,j,t-\tau_{ij}}^{\nu} - \sum_{k \in \Gamma^{\nu+}(j)} f_{i,j,k,t}^{\nu}, \quad \forall (i,j) \in E^{\nu}$$
(1)

$$\sum_{k\in\Gamma^{\nu+}(j)}q_{i,j,k,t}^{\nu}\leqslant S_{i,j,t}^{\nu},\quad\forall(i,j)\in E^{\nu}$$
(2)

$$f_{ij,k,t}^{\nu} \leqslant S_{ij,k,t}^{\nu} \ast b_{ij,k,t}^{\nu}, \quad \forall (i,j) \in E^{\nu}, k \in \Gamma^{\nu+}(j)$$
(3)

$$f_{i,j,k,t}^{\nu} \leqslant q_{i,j,k,t}^{\nu}, \quad \forall (i,j) \in E^{\nu}, k \in \Gamma^{\nu+}(j).$$

$$\tag{4}$$

Eqs. (1)-(4) mainly reflect vehicle flow conservation and their capacity restraints. Constraint (1) computes the available vehicle flow at a given time step for outgoing links based on the available vehicle flow, and the inflow to the link and the outflow to outgoing links at previous time step. Constraint (2) sets the available vehicle flows to be less than the maximum link capacity. Constraint (3) limits the outgoing flow to be less than the saturation flow rate at a given time, based on the signal status of the corresponding movement. Constraint (4) is to ensure that the actual outgoing flows will not exceed the available flow at that time.

4.3. Constraints for pedestrian flows in the pedestrian network

$$\sum_{k \in \Gamma^{p+}(j)} q_{i,j,k,t}^p = \sum_{k \in \Gamma^{p+}(j)} q_{i,j,k,t-1}^p + \sum_{k \in \Gamma^{p-}(i)} f_{k,i,j,t-\tau_{ij}}^p - \sum_{k \in \Gamma^{p+}(j)} f_{i,j,k,t}^p, \quad \forall (i,j) \in E^p$$
(5)

$$\sum_{k\in\Gamma^{p^+}(i)} q^p_{ij,k,t} \leqslant S^p_{ij,t} * b^p_{ij}, \quad \forall (i,j) \in E^p$$
(6)

$$\sum_{k\in\Gamma^{p+}(i)} f^p_{ij,k,t} \leqslant S^p_{ij,t} * b^p_{ij,t}, \quad \forall (i,j) \in E^p$$

$$\tag{7}$$

$$\sum_{k\in\Gamma^{p+}(j)} f^p_{ij,k,t} \leqslant S^p_{i,j,t} * b^p_{i,j,t}, \quad \forall (i,j) \in E^p$$
(8)

$$f_{i,j,k,t}^{\nu} \leqslant q_{i,j,k,t}^{\nu}, \quad \forall (i,j) \in E^{p}, k \in \Gamma^{p+}(j)$$

$$\tag{9}$$

$$b_{i,j}^p + b_{j,i}^p = 1, \quad \forall (i,j) \in E^p \tag{10}$$

$$b_{i,i,t}^p \leqslant b_{i,j}^p, \quad \forall (i,j) \in E^p \tag{11}$$

The pedestrian flow constraints are similar to the vehicle flow, where Constraints (7) and (8) set the upper-bounds of the total link inflows and outflows as the saturation flow rate. Constraint (10) enforces a unique direction flow for each pedestrian link. Constraint (11) guarantees that no pedestrian flow will exist in the prohibited direction.

4.4. Constraints for connection links

Eq. (12) is designed to reflect the process where evacuees move into vehicles and join the queue to the assigned safe destinations. The pedestrian flows are modeled to move for τ time steps (τ is integer) on the connection links before they are converted to vehicle flows. The vehicle flows will be converted from the pedestrian stream according to the car-pooling ratio λ . In urban networks, each connection node at the end of a connection link could be a bus stop or a parking lot. The ratio can be determined by the average car-pooling ratio obtained by sampling surveys for parking areas or by the bus holding capacities for transit pick-up points. For bus stops, a holding capacity constraint (13) is specified to prevent the total waiting passengers from exceeding the limit.

$$\sum_{k\in\Gamma^{\nu+}(j)} q^{\nu}_{i,j,k,t-1} = \lambda_i \sum_{k\in\Gamma^{\nu+}(j)} q^{\nu}_{i,j,k,t-1}, \quad \forall (i,j) \in E^c$$

$$\tag{12}$$

$$\sum_{k\in\Gamma^{\nu+}(j)}q_{i,j,k,t}^{p}\leqslant U_{i}^{p},\quad\forall(i,j)\in E^{c}$$
(13)

4.5. Constraints for dynamic signal controllers

There are either pre-timed or dynamic signal controllers at most urban intersections. Dynamic signal controllers account for the dynamic nature of the traffic network and are able to extend the green time of each phase based on the real-time detected traffic information. Controllers with the embedded function are able to adjust the signal timings on the basis of actual flows under the maximum and minimum green time constraints. Although not as flexible as the dynamic controllers, pre-timed signals are still the most widely deployed control at most intersections due to the concern of cost. Different from previous network-wide signal optimization formulations (e.g. Ziliaskopoulos, 2000), the proposed model incorporates the

safety concern of pedestrian movements into the signal phase sequence, and explicitly takes the lost time for each phase into consideration so as to prevent frequent phase transitions. The constraints related to the dynamic traffic signal controllers are set as follows:

$$\sum_{j} \nu_{ij,t} = 1, \quad \forall i \in V^{s}, \forall j \in C_{i}$$
(14)

$$\sum_{i} w_{i,l,t} = v_{i,j,t}, \quad \forall i \in V^s, \forall j \in C_i$$
(15)

$$w_{i,j,t} \ge w_{i,j-1,t-1} - w_{i,j-1,t} - 0.9, \quad \forall i \in V^s, \forall j \in C_i, j > 1$$
(16)

$$\sum_{\tau=t}^{+C_{ij}^{\max}+1} w_{ij,\tau} \leqslant G_{ij}^{\max}, \quad \forall i \in V^s, \forall j \in C_i$$

$$\tag{17}$$

$$\sum_{\tau=t}^{t+G_{ij}^{\min}} w_{ij,\tau} \leqslant G_{ij}^{\min} * (w_{ij,t} - w_{ij,t-1}), \quad \forall i \in V^s, \forall j \in C_i$$

$$(18)$$

$$b_{klmr}^{\nu} = w_{i,i,t}, \quad \forall i \in V^{s}, \forall j \in C_{i}, \forall (k,l,m) \in C_{i,j}$$

$$\tag{19}$$

$$b_{k,l,t}^{p} = w_{i,j,t}, \quad \forall i \in V^{s}, \forall j \in C_{i}, \forall (k,l) \in C_{i,j}$$

$$\tag{20}$$

The binary variable $v_{i,j,t}$ is defined to determine whether the signal is active for the *j*th barrier of intersection *i* at time *t*. For a typical 4-leg signal phase illustrated above, there are two barriers as indicated in the ring-and-barrier diagram. If any signal phases at the 1st barrier for the east-west streets is green, then set $v_{i,1,t} = 1$, otherwise $v_{i,1,t} = 0$. A similar variable $v_{i,2,t}$ is designed to determine whether any phases are green at the 2nd barrier for the North–South Street. Constraint (14) ensures only one barrier is active at each time. Constraint (15) states that if a barrier is active at a particular time, there is one and only one green phase belonging to the barrier in each ring. For example, if the left barrier is active, then only one of those phases (i.e. phases 1–6) in the upper ring and only one of the phases (13–18) in the lower ring will be at the green status. Constraint (16) guarantees that a yellow and all-red phase will follow immediately after each green phase. For example, phase 6 should be activated right after phase 5. Constraint (17) limits the phase green time to be under a specified maximum. Constraint (18) enforces a minimum green time to each phase. For the yellow and all-red interval phases (i.e., phases 2, 4, 6, ...), this study sets their minimum and maximum times at identical pre-determined values. Constraints (19) and (20) build the connections between the signals and the networks.

4.6. Constraints for pre-timed signal controllers

The pre-timed signal control constraints share the same constraints (14)-(16), and (19) and (20) as with the dynamic control. However, additional constraints are needed to determine the cycle length, phase sequences, and phase green times. Hence, the proposed model employs three new indication variables $u_{i,j,k,t}$, $v_{i,j,k,t}$ and $w_{i,j,k,t}$, where $w_{i,j,k,t}$ indicates whether current time *t* is within phase *k* of the *j*th cycle; $u_{i,j,k,t}$ and $v_{i,j,k,t}$ are used to determine $w_{i,j,k,t}$. All additional constraints for pre-timed controllers are listed below:

$$c_i = \sum_{j \in C_i} g_{ij}, \quad \forall i \in V^s$$
(21)

$$C_{i\min} \leqslant c_i \leqslant C_{i\max}, \quad \forall i \in V^s$$

$$g_{ii} = e_{i,k,i} - s_{i,k,i}, \quad \forall i \in V^{\mathrm{s}}, \forall j \in C_i, \forall k \leq K$$

$$\tag{23}$$

$$G_{ij,\min} \leq g_{ij} \leq G_{ij,\max}, \quad \forall i \in V^s, \forall j \in C_i$$
(24)

$$s_{i,j,k} + c_i = s_{i,j,k+1}, \quad \forall i \in V^s, \forall j, \forall k \in C_i$$

$$(25)$$

$$\boldsymbol{e}_{ij,k} + \boldsymbol{c}_i = \boldsymbol{e}_{ij,k+1}, \quad \forall i \in V^s, \forall j, \forall k \in \boldsymbol{C}_i$$
(26)

$$s_{i,j,k+1} = e_{i,j,k} + 1, \quad \forall i \in V^{s}, \forall j, \forall k \in C_{i}$$

$$(27)$$

$$-M\nu_{i,j,k,t} \leqslant t - e_{i,j,k} \leqslant M(1 - \nu_{i,j,k,t}), \quad \forall i \in V^s, \forall j, \forall k \in C_i, \forall t \leqslant T$$

$$\tag{28}$$

$$-Mu_{i,j,k,t} \leqslant s_{i,j,k} - t \leqslant M(1 - u_{i,j,k,t}), \quad \forall i \in V^s, \forall j, \forall k \in C_i, \forall t \leqslant T$$

$$\tag{29}$$

$$2w_{ij,k,t} \leqslant v_{ij,k,t} + u_{ij,k,t}, \quad \forall i \in V^s, \forall j, \forall k \in C_i, \forall t \leqslant T$$

$$(30)$$

$$w_{ij,t} = \sum_{k \in C_i} w_{ij,k,t}, \quad \forall i \in V^s, \forall j, \forall t \leqslant T$$
(31)

The above constraints are specified to regulate the controllers such that the signal can operate on a fixed cycle with constant green phases and time-invariant offsets. Constraint (21) computes the cycle length and Constraint (22) imposes a minimum and maximum on the cycle length. Constraint (23) computes the phase duration by measuring the time difference between a phase starting and its ending times. Constraint (24) also imposes restrictions on the minimum and maximum intervals. Constraints (25) and (26) ensure that the starting time difference for the same phase over consecutive cycles equals the cycle length. Constraint (27) regulates the phase sequence by setting the starting time of the next phase immediately after the ending time of the previous phase. Constraints (28) and (29) show how $u_{i,j,k,t}$ are determined by evaluating whether current time is within the green duration of a particular phase. Constraints (30) and (31) are used to determine $w_{i,j,k,t}$ based on $u_{i,j,k,t}$ and $v_{i,j,k,t}$.

4.7. System-optimal objective function

The common purposes for imposing traffic control and management on a congested network are to: (1) send most travelers to the safety area out of the danger zone within a given time window; (2) reduce the total cost for all the travelers (e.g. travel-time cost); and (3) prevent any lane blockage or spillback. The objective function for this research is for the first purpose, which is to maximize the total system throughput, i.e., the number of evacuees, during a time window. One can easily reformulate the objective function to fit the second and third purposes.

Note that since different types of vehicles can accommodate a different number of evacuees, formulating the objective function to satisfy the network conservation properties is not a straight-forward task. This study proposes to expand the mixed-flow network and reformulate it based on the new network. The new formulation can give the identical optimal objective to the original formulation.

First, one needs to group the connection nodes according to their carpooling rate, and then make a copy of the vehicle network for each group by setting the link capacity to $\lambda_i - 1$ times of the original network, where λ_i stands for the car pooling rate of group *i*, and attach the connection nodes to the duplicated vehicle network.

Fig. 4(a) presents an illustrative example, where the set of white, shaded, and black nodes represents pedestrian, connection, and vehicle nodes, respectively. The number on each link represents the roadway capacity. The first connection node has a carpooling rate of 50 (e.g., a bus stop), and the second has a carpooling rate of 2 (e.g. a passenger car parking lot). The new expanded network is constructed in Fig. 4(b), which consists of two vehicle networks as circled. The copy-1 network corresponds to a car-pooling rate of 50, so the capacities of these links in the newly created network are 49 times of their original values. The destinations are all connected to a super sink node (Hamacher & Tjandra 2002) with an infinite capacity.

The above expanded network can hold the flow conservation relation at all connection nodes. By doing so, one can generate a network with its link flows representing the actual number of evacuees under different types of evacuation vehicles. Constraint (12) for connection links needs to be replaced with Constraint (32).

$$\sum_{k \in \Gamma^{\nu+}(j)} q_{i,j,k,t-1}^{\nu} = \sum_{k \in \Gamma^{\nu+}(j)} q_{i,j,k,t-1}^{\nu}, \quad \forall (i,j) \in E^{c}$$
(32)

By denoting i_n as the corresponding node in the replicated network n for node i in the original network, constraints (33) and (34) guarantee that the signal setting at the intersection of the extended vehicle network is identical to the original network.

....

$$b_{i,j,k,t}^{\nu} = b_{i,j,n,k,n,t}^{\nu}, \quad \forall (i,j) \in E^{\nu}, \forall k \in \Gamma^{\nu+}, \forall t \leqslant T, \forall n \in C^{V^{*}}$$

$$(33)$$

$$b_{i,j,t}^{p} = b_{i,j,t}^{p}, \quad \forall (i,j) \in E^{p}, \forall t \leq T, \forall n \in C^{V^{c}}$$

$$(34)$$

The objective function for this research subject is to maximize the total evacuee throughput to all the pre-designated destination nodes within a given time window. The objective function can be expressed with Eq. (35). Note that k = -1 indicates j is a destination node, similarly, i = -1 indicates j is a origin node

$$\sum_{i\in\Gamma^{\nu-}(j)}\sum_{j\in D}q_{i,j,-1,T}^{\nu}$$
(35)

However, the expanded network formulation may yield unrealistic optimal flows. For instance, in Fig. 4(c), there are two chains γ_1 and γ_2 with the flow values of 500 and 1000 in these two replicated networks; however, no flow exists in the original network. The restructured feasible flow distribution that is consistent to their original patterns is shown in Fig. 4(d). The total pedestrian throughputs to the final destinations are identical. The only difference is the distribution of the link flows on the network. Constraints (36) and (37) are proposed for such a need.



Fig. 4. Network expansions for objective formulation.

$$q_{ij,k,t}^{\nu} = \sum_{n} \left(\frac{1}{\lambda_n - 1} q_{i_n, j_n, k_n, t}^{\nu} \right), \quad \forall (i, j) \in E_o^{\nu}, \forall k \in \Gamma^{\nu_+}, \forall t \leqslant T, \forall n \in C^{V^c}$$

$$(36)$$

$$f_{ij,k,t}^{\nu} = \sum_{n} \left(\frac{1}{\lambda_n - 1} f_{i_n j_n, k_n, t}^{\nu} \right), \forall (i, j) \in E_o^{\nu}, \forall k \in \Gamma^{\nu_+}, \forall t \leqslant T, \forall n \in \mathbb{C}^{V^c}$$

$$(37)$$

where E_o^{ν} is the link set in the original network; C^{ν^c} is the set of connection node groups categorized by the car-pooling rate; i_n is the node in the *n*th copied network corresponding to original node *i*.

To help readers to understand the above two equations better, let's take Fig. 4(c) and (d) for example. In Fig. 4(c), the flows on the connection links of node 1 (dotted arrow lines in the origin and Copy 1 network) violate Constraint (37) since $0 \neq \frac{1}{\lambda_1 - 1} * \gamma_1 = \frac{1}{50 - 1} * 500$. So is the case for flows on the connection links of node 2, since the flow in the original network is always 0. However; in Fig. 4(d), Constraint (37) is satisfied for connection links of both nodes since $10 = \gamma'_1 = \frac{1}{\lambda_1 - 1} * \gamma_1^* = \frac{1}{50 - 1} * 490 = 10$ and $500 = \gamma'_2 = \frac{1}{\lambda_2 - 1} * \gamma_2^* = \frac{1}{2 - 1} * 500 = 500$. Thus Constraints (36) and (37) can rule out the infeasible flow solutions in the original network.

The above formulation can be solved by the commercial OR solvers (e.g. CPLEX). IBM ILOG CPLEX Optimizer provides flexible, high-performance mathematical programming solvers for linear programming, mixed integer programming, quadratic

Table 1		
Optimized	signal	timings.

Cycle	Phase 1 (s)	Phase 3 (s)	Phase 5 (s)	Phase 7 (s)	Phase 9 (s)	Phase 11 (s)
(a) Dynamic signal	controller					
1st	44	15	5	5	30	15
2nd	54	22	8	12	30	22
3rd	52	28	10	14	22	28
Bottom 3rd	64	90	20	25	5	90
Bottom 2rd	63	90	20	24	5	90
Bottom 1st	60	90	20	21	5	90
Cycle length (s)			Phase			Green time (s)
(b) Pre-timed signal controller						
139			1			68
			2			4
			3			56
			4			4
			5			20
			6			4
			7			24
			8			4
			9			16
			10			4
			11			56
			12			4
(c) conventional pr	e-timed signal controll	ler-1				
50			1			32
			2			24
			3			16
			4			16
			5			24

programming, and quadratically constrained programming problems. The model we proposed here belongs to the category of the mixed integer programming, thus CPLEX is an ideal commercial solver for our model. To further improve the computational efficiency and apply it into large-scale networks, we also devised an efficient algorithm based on the Bender's Decomposition method. However; due to the page limitation, we will not discuss it here in detail.

5. Numerical example

This section presents an application of the proposed model for evacuation planning at the M&T stadium in the Baltimore downtown.

5.1. Evacuation scenario

The scenario assumes that a total of 20,000 individuals need to leave the stadium after the football game. The available transportation modes include private cars, buses and the light rail. The layout of the M&T stadium is depicted in Fig. 5(a).

The pedestrian network is shown in Fig. 5(b). The source node 1000 represents the stadium; the sink nodes 101–105 are the parking lots; and the sink node 106 is the pick-up point for those without access to passenger cars. The solid arrow lines are sidewalks and the dashed arrow lines are crosswalks.

The pedestrian destination nodes are connected to the vehicle source nodes, which are also the connection nodes. The capacity of each connection nodes are estimated based on the available parking or waiting spaces and the average car access times are approximated based on the size of each parking lot. The carpooling rates are assumed to be 2 for the parking lots and 20 for the transit stops.

The vehicle network is presented in Fig. 5(c). Those arrows indicate the possible flow directions between nodes. The sink nodes 301, 302, 303, 304 and 305 can be viewed as destinations for vehicles to further access I83, US40, I395 South, MD295 South, and MD2, respectively. These solid lines represent the vehicle roads and the other lines denote the connectors or the turning movements. The Dashed lines represent intersection links which are subjected to conflicts.

The crossing of Hamburg St and MD 295 (Fig. 1) is a critical intersection with the most potential conflicts. The ring and barrier diagram for this intersection is depicted in Fig. 5(d) (only include possible turning movements for the stadium traffic). Both the pre-timed and dynamic signal controllers have been tested with our model.



(a) Layout of the M&T stadium in Baltimore city



Fig. 5. Scenario setting and model input.

5.2. Optimized results

The proposed control strategies generated from the optimization model include:

- (1) Directions of pedestrian flow movement;
- (2) Signal timings for each phase (dynamic control or pre-timed control);



Fig. 6. Generated flow distributions in the network.

- (3) Time-varying flows on each pedestrian walkway;
- (4) Time-varying flows on each vehicle roadway;
- (5) Arriving rate of pedestrian flows at the parking areas and transit stop; and
- (6) Vehicle and evacuee flows to each final vehicle destination.

Table 1(a) lists the optimized signal timing plan for the dynamic controller. Since the timings are different from cycle to cycle, only the first and last 3 cycles are shown in the table. In comparison, the optimized signal timing plan for the pretimed controller is listed in Table 1(b). Fig. 6(a) and (b) shows the optimized flow distribution aggregated over five minutes in the pedestrian network at the start of the evacuation and at the end of the first hour, respectively. It can be observed that more pedestrian flows are assigned to the network at the beginning period since there are relatively fewer vehicle flows during the initial phase. Fig. 6(b) also shows the hourly pedestrian throughput for each destination. Fig. 6(c) and (d) shows the optimized flow distribution aggregated over five minutes in the vehicle network at the start of the evacuation and at the end of the first hour, respectively. Notably less vehicle flows are assigned to the network at the starting period than 1 h later since the evacuees need more times to access their vehicles in the beginning period. Fig. 6(b) also shows the hourly vehicle throughput for each destination.

5.3. Model comparison

In this section, we compare the differences by adopting different types of signal controllers: dynamic and pre-timed control. It is noticeable that green times for the pedestrian phase (phase 9) are relatively long at the beginning period (i.e., 30 s) and then gradually reduced to a shorter interval (i.e., 5 s). This is consistent with reality that there are more crossing evacuees and fewer vehicles in the network at the beginning of the evacuation, thus demanding longer green times for the pedestrian phase so that more people can access their vehicles or get to the pick-up points. In comparison, the fixed green time for each phase in the pre-timed controller sits moderately between the shortest and longest values for the corresponding phase generated by the dynamic controller. By comparing the throughput curves under the dynamic control and the pretimed control in Fig. 7(a), one can see that the benefits of dynamic control are quite clear during the early stage of evacuation. This is likely due to the timings allocated for the vehicle movements are not fully utilized under the pre-timed controllers at the beginning period. As time goes by, the relative benefits obtained by the dynamic control begin to decrease. It is mainly due to the fact that the traffic conditions on the network are becoming congested, and the adaptive control will function like a pre-timed one under over-saturated conditions. Conceivably, dynamic control will exhibit the benefits at the later stage of the evacuation by reducing the green time of the pedestrian walking phase where most pedestrians have gained access to their vehicles.

In our proposed optimization model, we also have made the following three major modifications to the conventional methods:



Fig. 7. Model comparisons.

Table 2

Comparison of evacuation results under different objectives.

Objective function	Evacuee throughput	Vehicle throughput	Percentage of buses (%)	Percentage of evacuees in buses (%)
1 (Maximize Evacuees)	5234 persons	2077 vehicles	2.9	23
2 (Maximize Vehicles)	4868 persons	2389 vehicles	0.2	2.0

• Using a special phase to separate crossing pedestrians from right-turn vehicles;

• Taking the transition time between the phases into consideration to minimize loss time; and

• Formulating the objective as maximizing the evacuee throughput to account for different types of evacuation vehicles

To illustrate the importance of the first two model modifications, we have re-executed the model and generated the optimized signal timing plans with the typical signal phasing plan shown in Fig. 5(e). Table 1(c) shows the results that the optimal evacuation model under the typical phasing plan will yield much shorter signal timings which are mostly insufficient for the need of evacuation flows. Another notable property is that without considering the potential conflicts between the massive pedestrian crossings and right-turn vehicles, any optimal evacuation model may substantially overestimate its efficiency and yield unreasonable flow patterns. Fig. 7(b) depicts two ratios: the ratio between the right-turn movement flow from and its saturation flow rate, and the ratio between the conflicting pedestrian flow and its saturation flow in each cycle. It is expected that the sum of these two ratios (i.e. volume to capacity ratios) should not exceed 1 since these two movements are in direct conflict and only one movement is allowed at a particular time interval. However, both ratios, without accounting for the pedestrian–vehicle conflicts, are close to 1 which implies these two flows coexist during most of the evacuation time except at the beginning stage when no vehicles are loaded onto the network. This is obviously, unrealistic and evidences the need to address such conflicts. In brief the formulations based on conventional phase plans tends to give unrealistic flow patterns, over-estimate the objective function, and generates short green durations.

Another important contribution of this model is to formulate the objective function as maximizing the evacuee throughput rather than the vehicle throughput to each destination by creating the expanded mixed-flow network. If the objective function is to maximize the total number of vehicles arriving at the safety nodes over a given time window, one needs not to expand the network. Table 2 compares the throughput results generated from the two objective functions. The percentage of buses is the proportion of buses in the total arriving vehicles to the destination nodes, and the percentage of evacuees in buses is the proportion of evacuee throughput rescued by buses.

It is evident that

- Targeting evacuation on maximizing vehicles or evacuees can lead to totally different control strategies and the outcomes as shown in the evacuee and vehicle throughputs.
- As reflected in the percentage of buses, the control plan for Objective-1 evacuate more evacuees using the transit vehicles, while the control plan with Objective-2 favors those with their passenger cars.
- Under the first objective, the small proportion of buses (only less than 3%) play a critical role in evacuating a significant number of people (more than 20%) to those destination nodes.

6. Summary and conclusion

This paper has presented an enhanced model for integrating the optimization of the mixed-flow movements and the signal timings within the metropolitan area that takes into account the conflicts between congested vehicle and pedestrian flows. The proposed model employs the common node-link concept to represent the pedestrian and vehicle networks, and designs connectors to model turning movements at intersections. The connection node is defined to connect the flow conversion between pedestrian and vehicle flows. Based on the locations of pick-up points for evacuees using transit systems and parking garage distributions for those having access to vehicles, the proposed model is capable of producing the set of effective routing strategies to guide pedestrians and vehicles, and also provide the optimal signal timings for either the dynamic or pre-timed signal controls. A set of constraints representing signal control mechanisms are employed in the model to capture the interactions between pedestrians and vehicles in the metropolitan area. An illustrative example presented in the paper seems to indicate that the promising properties of the proposed model. The results of simulation experiments clearly indicate that a failure to account for potential mixed-flow conflicts will yield unrealistic evacuation plans and overestimate the operating efficiency.

The model presented in this paper can be applied to planning and operations of evacuation scenarios under recurring and planned events, such as football games, celebration events and even hurricane hazards, some input parameters for this model, such as walking facility layouts and car pooling rates, need to be known in advance. In addition, this model can also serve as the purpose of evacuation planning for unplanned and no-notice events, such as terrorist attacks at critical infrastructures. In this case, user can create hypothetical scenarios including the magnitude of the incident, the impact areas and the safety destinations, and then run the model to generate optimal plans and evaluations on the plan for better preparation and training for the incidents. Although the nature of the real-world evacuation is very complicated and unpredictable, our model can

provide a best-case reference to the planners, assuming the optimized plan can be reinforced effectively. More often than not in evacuations, pedestrian rushes onto street in a burst mode, resulting in a severe blocking of the street. In fact, even if pedestrians can access the connection points in no time, if the vehicle flow cannot evacuate in a timely manner, they still have to queue at that point and the long queue may deteriorate the entire traffic condition. The dynamic optimized network flows generated by the model not only provides the underlying basis for the intersection control mechanisms, but also act as a guideline for reinforcement teams at critical walking facilities to maintain pedestrian order and control flows. The optimized signal timings can be pre-coded into the controllers and override the normal plans upon evacuations, or it can provide the traffic enforcement teams a quantitative guideline on how to coordinate the traffics manually at critical intersections.

Note that this work is exploratory in nature, and we fully recognize that much remains to be done in developing an efficient and operational system that can guide pedestrians to the most proper mode, direct various types of traffic flows to the most efficient routes, and set the most proper signal timing. Our ongoing research along this line is to explore various structures of the proposed formulations, and to develop a more efficient network algorithm. A more detailed simulation-based microscopic approach to simulate the optimized mixed-flow patterns is also one of our on-going research subjects.

Appendix A

In view of the complex mixed flow nature and the intended use of the maximum flow algorithm for computing efficiency, this paper has designed an innovative network conversion method which is described below with an illustrative example.

Fig. A-1 illustrate an example mixed flow network, where the set of white, shaded, and black nodes represents pedestrian, connection, and vehicle nodes, respectively. The number with a bracket under each link represents its separate flow capacity. The two connection nodes are labeled 1 and 2, respectively. A parameter λ indicating the carpooling rate is associated with each connection as depicted in the figure. The connection node 1 has a carpooling rate of 50 persons per car (e.g., a bus stop), and the connection node 2 has a carpooling rate of 2 persons per car (e.g. a passenger car parking lot). In typical network solution algorithms, e.g. maximum flow problem, the actual flow on each link is treated as a decision variable and the objective is to maximize the total flow entering the sink node (the black vehicle node). However; in this study our objective is to evacuate as many pedestrians as possible, not the vehicle flows. In addition, typical network solution algorithm requires the flow conservation property at each every node, however; the connection nodes do not satisfy the property since fewer flows will exit the nodes due to the carpooling rate parameter.

To overcome the above two shortcomings, we need to expand the network by duplicating another vehicle network for each connection node. The expanded network is depicted in Fig. A-2, and the link capacities are set as depicted. The classic network flow algorithm can be applied on this expanded network because the flow conservation and the maximization of pedestrian flows are satisfied. The capacities of the links in the expanded network are set to be (carpooling ratio – 1) times those in the original network. For example, the capacity of the link from the connection node 2 to the vehicle node in the 1st expanded network is $(50 - 1)^* 1500 = 73,500$.

However; there are some side effects by simply running the classic network flow algorithm on this expanded network. Fig. A-3 gives an example of a possible optimal flow solution. All the flows (γ_1 and γ_2) go through the replicated networks and no actual flows exist on the real-world road links. In addition, there are no easy ways to convert the solution to reflect the meaningful flow patterns on the original network. Taking one step back in finding where the issues come from, it can be observed that the objective value (total flow throughput to the destination node) is already optimal but the flow patterns are incorrect.

To prevent the above issue from happening, additional constraints (36) and (37) need to be added in our model. The purpose of these constraints is to regulate the flow patterns to reality under the same optimal objectives. The ratio of the flow on each link in the original network to the corresponding link in the expanded network should be equal to the ratio of their capacities. As an example, readers can check the flow patterns are no longer valid under the newly added constraints. In Fig. A-3, the flows on the connection links of node 1 (dotted arrow lines in the origin and Copy 1 network) violate Constraint (37) since $0 \neq \frac{1}{\lambda_1 - 1} * \gamma_1 = \frac{1}{50 - 1} * 500$. So is the case for flows on the connection links of node 2, since the flow in the original network is always 0.

Readers can also verify the intended optimal flow pattern depicted in Fig. A-4 satisfies the constraints since

$$10 = \gamma_1' = \frac{1}{\lambda_1 - 1} * \gamma_1^* = \frac{1}{50 - 1} * 490 = 10$$

And

$$500 = \gamma'_2 = \frac{1}{\lambda_2 - 1} * \gamma^*_2 = \frac{1}{2 - 1} * 500 = 500.$$



Fig. A.

Thus Constraints (36) and (37) are crucial in ruling out the infeasible flow solutions while preserving the intended ones in the original network.

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