Integrated Off-ramp Control Model

Zichuan Li, Gang-Len Chang, and Suhasini Natarajan

Abstract— **Ramp metering is a very important method to improve freeway traffic system performance, but current researches emphasize only on the entering flow control, and not much attention has been paid to the exiting volume at off-ramp. In some situations, the exiting queue will decrease the overall system performance by a large magnitude. This is demonstrated by simulation experiments and field observation. In this study, a mix integer model is proposed to optimize the arterial and off-ramp control signal timing based on cell transmission traffic propagation model. A Genetic Algorithm based solution algorithm is proposed along with a numerical case study to demonstrate the benefits of this model.**

I. Introduction

Nowadays, highway system plays a more and more important role in daily social life. As traffic increases day by day, recurrent traffic congestions emerge as a more and more serious problem. To fight with these congestions, various control strategies are employed to overall ameliorate the freeway traffic condition, such as speed control, ramp metering (on-ramp metering), etc. Among these methods, ramp metering is widely employed because of its ease of implementation.

There is a large body of literature talking about on-ramp metering. And typically, there are two major categories of control strategies for ramp metering, i.e., local control and system control. The former category includes various algorithms, which are based on empirical analysis [17] and automatic control theory [6], [8] and [14]. An overview of ramp-metering algorithm, including the early fixed-time approaches to traffic-responsive rules and modern sophisticated nonlinear optimal-control schemes can be found in the study of Papageorgiou and Kotsialos [15].

Wattleworth and Berry [16] made the first attempt to optimize the ramp metering control at the system level. This paper and its followers in [11] propose a time-invariant linear program to minimize the total travel time for the entire freeway system. These models assume that freeways operate under free speed; the time-dependent origin-destination (OD) demand information is available; and there are no diversions from freeways to surface arterial street. In 2000, Lovell and Daganzo [10] extended Wattleworth's model to include time-dependency by employing departure curves and assumed no exit spill-over.

To model the traffic dynamics of the freeway, macroscopic traffic flow models have been employed and combined with optimization theory to obtain optimal control strategies. Papageorgiou [13] proposed a linear optimal-control model including both motorways and signal-controlled urban roads based on the store-and-forward modeling philosophy. Zhang and Recker [18] analyzed the state and control relationships, and obtain some general analytical results based on the well-known Lighthill-Witham-Richard (LWR) model. Chang and Li [2] constructed a linear dynamic model with a quadratic objective function for integrated-responsive ramp-metering control based on Payne's continuum traffic stream model. Kotsialos and Papageorgiou et al. [9] considered ramp metering, route guidance, and motorway-to-motorway control measures simultaneously in a discrete-time optimal control problem based on the METANET [12] and METACOR [5]. Zhang and Levinson [19] proposed an analytical framework for ramp metering whose input variables are all directly measurable by detectors in real-time. Gomes and Horowitz [7] proposed a nonlinear optimization problem for the on-ramp metering control problem by utilizing the asymmetric cell transmission-link model (ACTM) and solve a simple linear version with certain constraints applied.

But none of those models deal with the off-ramp control. In these models, off-ramps are considered as a traffic sink, i.e., the vehicles arriving at the off-ramp will be removed from the system immediately. But in real world, this is not always true. As described by Lovell [11], since the drivers tend not to segregate themselves by destination in advance of an off-ramp, but rather make most of their lane-changing decisions at the last second, the exit queue of an off-ramp might spread itself laterally upstream of an off-ramp, thereby restrict mainline flow as depicted in Fig. 1.

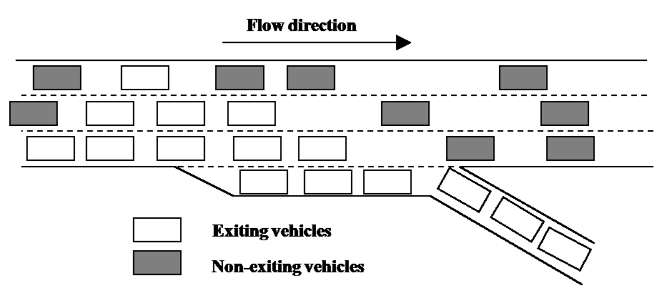

Fig. 1. Lateral spreading of an exiting queue upstream of an off-ramp.

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Zichuan Li is Research Assistant at Dept. of Civil and Environmental Engineering, University of Maryland at College Park, College Park, MD 20742 USA, (corresponding author) Phone: 301-405-2638, Fax: 301-405-2585, (e-mail: lizch@ umd.edu)

Gang-Len Chang is Professor at Dept. of Civil and Environmental Engineering, University of Maryland at College Park, College Park, MD 20742 USA (e-mail: gang@glue.umd.edu).

Suhasini Natarajan is Research Assistant at Dept. of Civil and Environmental Engineering, University of Maryland at College Park, College Park, MD 20742 USA, (e-mail: suhasini@ umd.edu)

To depict the above fact, a simulation study of the MD97@I495 interchange was conducted and its details are discussed in the remaining of this section. Fig. 2 is a sketch of the MD97@I495 interchange. In the morning peak hour, the off-ramp volume (off-ramp A) from I495 west bound to MD97 south bound is large as lot of people living in suburban area go to work and a long exiting queue is formed. Currently, the off-ramp from I495 west to MD97 (off-ramp A) is controlled by signal and the other two are controlled by yield signal. To evaluate the effect of exiting queue on upstream mainline traffic, the demand from each entering link is fixed and only the exit ratio at off-ramp A is increased from 30% to 40%.

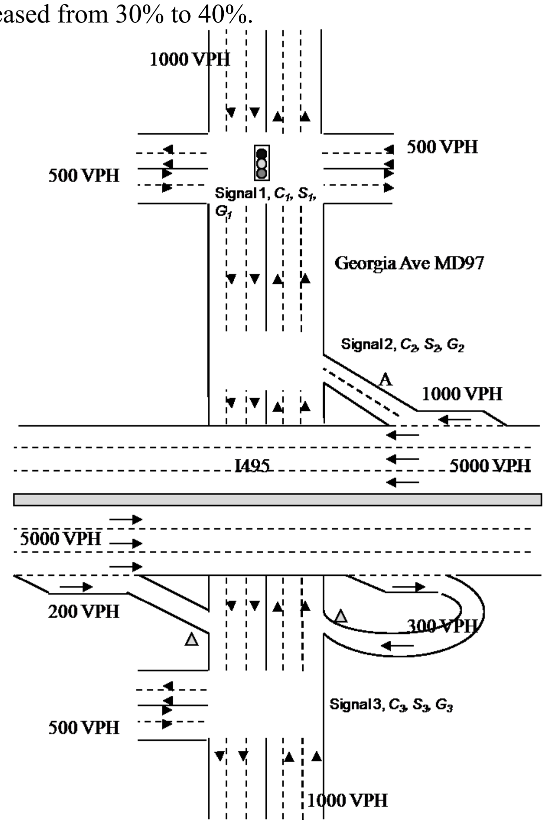

Fig. 2. MD97@I495 Interchange

The simulation results from CORSIM package are listed in Table 1, which indicates that the effect on upstream mainline traffic increases dramatically when off-ramp volume increases from 750 vphpl (30% exit ratio) to 1000 vphpl (40% exit ratio) in terms of average delay and average speed. There are two main reasons. One is the increment of lane-change number, and the other is the spillback of exiting queue.

TABLE I
OFF-RAMP QUEUE EFFECT ON
UPSTREAM MAINLINE TRAFFIC PERFORMANCE

Off-ramp Volume (vph)	Average Delay (s/veh)	Average Speed (mph)
1000	96.4	7.3
900	68.1	9.87
870	41.7	14.69
800	9.7	36.05
750	1.4	57.95

Anani [1] studied the exiting queue of an off-ramp by field data from video-tapes, and found that a bottleneck with a diminished capacity arises on a freeway segment whenever queues from the segment's off-ramp spilled-over and occupied its mandatory exit lane and the lengths of these exit queues are negatively corrected with the discharge flows in the freeways segment's adjacent lanes, i.e., longer exiting queues from over-saturated off-ramp are accompanied with lower discharge rates for the non-exiting vehicles. These findings are confirmed by the above simulation results.

So to benefit the entire system, it is necessary to consider the effect of off-ramp exiting queues on upstream mainline traffic when optimizing the signal timing of arterial near off-ramp, i.e., to obtain an optimal signal plan for freeway-arterial system or freeway entering ramp metering, the effect of exiting queue on upstream mainline traffic should be considered.

The remaining part of this paper is organized as follows. Section II will present the details of the model, its formulation and solution algorithm. A case study with results will be described in Section III and conclusions are presented in Section IV.

II. OFF RAMP CONTROL MODEL

A. Model overview

As discussed in Section I, to obtain the optimal signal plan for freeway-arterial system or freeway entering flow metering system, the effect of off-ramp exiting queue to upstream freeway traffic should be considered. Unfortunately, till now, no estimating model for this effect has been developed according to the literature. As currently no data are available to develop such a model, this study focuses on developing an off-ramp signal optimizing model for the local arterial and off-ramp traffic with the constraint that the off-ramp exiting queue cannot spill over, so as not to affect the upstream mainline traffic.

The model presented in the following subsections employs the cell transmission model [3], [4] to represent the traffic propagation and a mix integer program is formulated to find the optimal signal timing for the target control area.

B. Definition of basic traffic variables

Parameters:

τ: time step

d_i^t : *demand of cell i at time interval t*

s: exit cell set

x_i^t : *number of vehicles in cell i at time interval t*

y_{ik}^t : *number of vehicles moving from cell i to cell k at time interval t*

R_i^t : *receiving capacity of cell i at time interval t*

S_i^t : *sending capacity of cell i at time interval t*

$Q_i = saturation flow of the link \times \tau$: *saturation flow rate of cell i*

l: *cell length*

K: *saturation density*

$N_i^t = l \times K$: *holding capacity of cell i*

$\Gamma(i)$: *downstream cell set of cell i*

$\Gamma^-(i)$: *upstream cell set of cell i*

MinG: minimum green time
MinC: minimum cycle length
MaxC: maximum cycle length

$r_{kj}^t = \begin{cases} 1, \text{ if the } j^{th} \text{ movement of signal } k \text{ is green at time } t \\ 0, \text{ otherwise.} \end{cases}$

L_m : *maximum queue length allowed on link m*

$\psi(m)$: *cell set of link m*

$Q_{kj,}$: *adjusted saturation flow rate of* j^{th} *approach of signal k.*

S_{kj} : *sending capacity of* j^{th} *movement of signal k.*

$X_{kj,}$: *sending capacity of* j^{th} *movement of signal k.*

E_{kj}: *number of vehicles entering the last cell of signal k of approach j*

$ratio_{kj:}$ *turning ratio of* j^{th} *movement of signal k.*

Decision variables:

C_k : *cycle length of signal i*
S_k: *offset of signal k*
$g_{kj,}$: *green time of jth approach of signal k.*

Among those decision variables, the signal cycle length is dependent on the green time. So the independent decision variables include offset and green times only.

C. Model formulation

$$\text{Max} \sum_{t=1}^{T} \sum_{s} y_{\Gamma(s),s}^t \quad (1)$$

$$\text{Min } \tau \sum_{t=1}^{T} \sum_{i} (x_i^t - y_i^t) \quad (2)$$

The purpose of the objective function (1) is to minimize total delay of all the vehicles in the control area. Alternatively, this study also proposes a second objective function (2), which maximizes the total throughput and is suitable for over-saturation condition.

In this study, cell transmission model is employed to represent the traffic propagation, which employs two categories of constraints to capture the traffic propagation in the control area. The first category of constraints represents the traffic propagation on the links, and the other represents traffic dynamics of signalized intersections.

Among the link traffic propagation constraints, (3) represents the flow conservation law. Equation (4) is designed to represent the vehicles entering the network. Equations (5) and (6) state that the total number of vehicles entering and exiting a cell during an time interval cannot exceed its entering and sending capacity, which are defined in (7) and (8) respectively.

For signal control, NEMA coding method is employed to describe the split information (see Fig. 3). To represent the signal control strategy, at each time step, for movement *j* of signal *k*, a binary variable is defined to indicate whether or not the corresponding movement is green. Equations (9), (10), (11), (12) intend to make r_{kj}^t equal to 1 for green or 0 for red.

$$x_i^{t+1} = x_i^t + \sum_{k\in\Gamma(i)} y_{ki}^t - \sum_{j\in\Gamma^{-1}(i)} y_{ij}^t \quad (3)$$

$$x_r^{t+1} = x_r^t + d_r^t - \sum_{j\in\Gamma^{-1}(r)} y_{rj}^t \quad (4)$$

$$\sum_{k\in\Gamma(i)} y_{ki}^t \le R_i^t \quad (5)$$

$$\sum_{j\in\Gamma^{-1}(i)} y_{ij}^t \le S_i^t \quad (6)$$

$$R_i^t = \min\{Q_i^t, N_i^t - x_i^t\} \quad (7)$$

$$S_i^t = \min\{Q_i^t, \ N_i^t\} \quad (8)$$

$$\mathrm{G_{kj}} - \mathrm{mod}(t - S_k, \mathrm{C_k}) \le r_{kj}^t C_k, \ j = 1,2,3,6,7 \quad (9)$$

$$\mathrm{G_{kj}} - \mathrm{mod}(t - S_k, \mathrm{C_k}) \ge (r_{kj}^t - 1) C_k, \ j = 1,2,3,6,7 \quad (10)$$

$$\mathrm{mod}(t - S_k, \mathrm{C_k}) - \mathrm{G_{k,j-1}} \ge (r_{kj}^t - 1) C_k, \ j = 1,2,3,6,7 \quad (11)$$

$$\mathrm{mod}(t - S_k, \mathrm{C_k}) - \mathrm{G_{k,j-1}} \le r_{kj}^t C_k, \ j = 1,2,3,6,7 \quad (12)$$

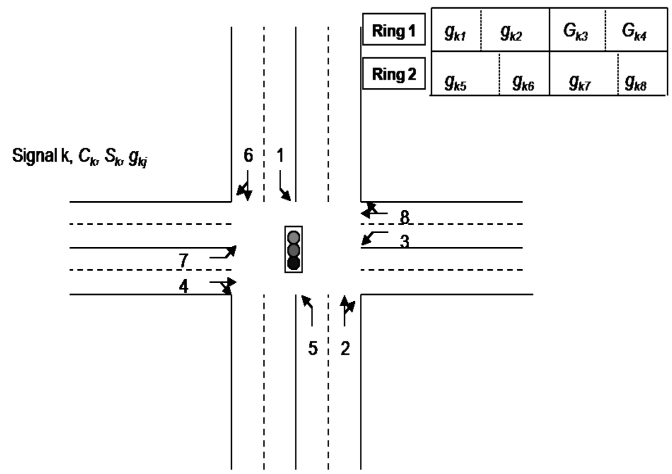

Fig. 3. NEMA coding for a signalized intersection

Equations (13), (14) state the existence of barrier, which means that the green time summation of phase 1 and phase 2 should be equal to that of phase 3 and phase 4. By (15), only one of the four phases, i.e., phase 1 to phase 4, is allowed to be green.

$$r_{k3}^t + r_{k4}^t = r_{k7}^t + r_{k8}^t \quad (13)$$

$$r_{k1}^t + r_{k2}^t = r_{k5}^t + r_{k6}^t \quad (14)$$

$$r_{k1}^t + r_{k2}^t + r_{k3}^t + r_{k4}^t = 1 \quad (15)$$

Equations (16) , (17) , (18) , (19) , (20) , (21) , (22) translate the split information into the time points in (9), (10), (11), (12).

$$G_{k0} = 0 \quad (16)$$

$$G_{k1} = g_{k1} \quad (17)$$

$$G_{k2} = g_{k1} + g_{k2} \quad (18)$$

$$G_{k3} = g_{k1} + g_{k2} + g_{k3} \quad (19)$$

$$G_{k5} = g_{k5} \quad (20)$$

$$G_{k6} = g_{k5} + g_{k6} \quad (21)$$

$$G_{k7} = g_{k5} + g_{k6} + g_{k7} \quad (22)$$

To apply the minimum green time, minimum cycle length and maximum cycle length constraints, constraints (23) and (24) are employed.

$$g_{jk} \geq MinG \tag{23}$$

$$MinC \leq C_j \leq MaxC \tag{24}$$

The flow relationship at a signalized intersection is more complicated than freeway junctions and additional information is needed, since several links of road sections are connected at a single point. So the concept of subcells is introduced to record the additional information.

At an approaching road section, vehicles can perform several movements. So the basic idea to model the signalized intersection is to divide the approaching cell (the cell corresponding to the approaching road section) into the same number of cells as its legal movements. For example, consider a typical four-leg signalized intersection as shown Fig. 3 The corresponding configuration with subcells is shown in Fig. 4 and details of the east bound are shown in Fig.5.

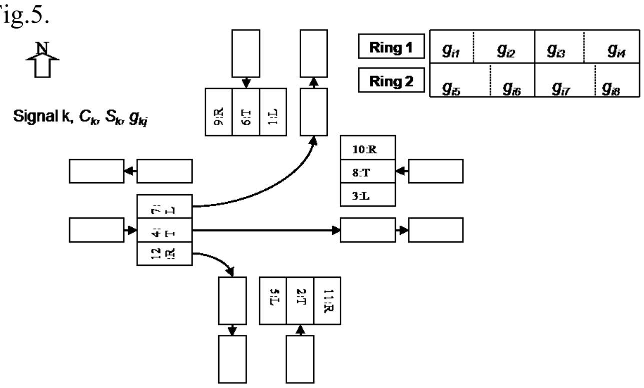

Fig. 4 Deployment of a four-leg intersection

Fig. 4 illustrates that a typical four-leg intersection can be represented by four diverging cells and four merging cells. Each approaching cell is divided into three subcells representing three movements. As shown in Fig. 5, cell i is the last cell of a link, and it is also the upstream cell for intersection k corresponding to movement 4, 7, 12 in NEMA coding, in which subcell 12 is for right turning movement, subcell 4 for through traffic and subcell 7 for the left turning movement. The variables for each subcell are defined as : $sx^t_{k,j}$ is the number of vehicles in subcell j, which can be calculated from the diverging proportion of each movement; $sy^t_{k,j}$ is the number of vehicles than can be discharged by sub-cell j in time t, and sq^t_{kj} is the saturation discharge rate of movement j in time t.

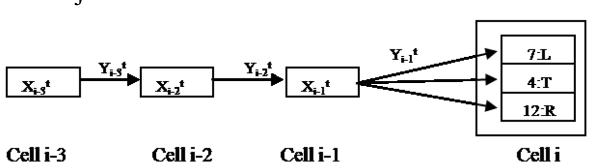

Fig. 5. Diverging cell connection for approaching road segment

The additional equations for the subcells are defined as:

$$sx^t_{i,k} = sx^{t-1}_{i,k} + Y^t_{i-1} \times ratio_{jk} \tag{25}$$

$$ss^t_{kj} = \min\{sq^t_{kj} \times r^t_{kj},\ sx^t_{i,k}\} \tag{26}$$

$$sy^t_{kj} = \min\{ss^t_{kj}, R^t_{kj}\} \tag{27}$$

Where $Y^t_{i-1}\, r^t_{i,k}$ is the number of vehicles in cell *i-1* going to subcell k of cell i, which is the product of the vehicle number in cell *i-1* and the corresponding diverging ratio; ss^t_{kj} is the maximum number of vehicles that can be sent by movement j to the corresponding downstream cell, and R^t_{kj} is the maximum number of vehicles allowed by downstream cell of movement k at time t, which is defined in (28). The number of vehicles that can move from subcell k of cell i to its downstream cell, $sy^t_{i,k,}$ is defined by (27).

Equation (28) indicates that the receiving capacity of a downstream cell is the adjusted saturation flow rate or the remaining space in the corresponding cell, whichever is lesser. Equation (29) states that the sending capacity of the entering cell is the smaller value of the through capacity and the vehicle number in the corresponding entering cell. Equation (30) describes that the vehicle movements from upstream to downstream in a time interval cannot exceed the sending capacity of upstream cell and the corresponding downstream receiving capacity.

$$R^t_{kj,d} = \min\{Q_{jk}, N^t_{kj,d} - x^t_{kj,d}\} \tag{28}$$

$$S^t_{kj,u} = \min\{Q_{jk} r^t_{kj}, x^t_{kj,u}\} \tag{29}$$

$$y^t_{kj} = \min\{R^t_{kj,d}, S^t_{kj,u}\} \tag{30}$$

To avoid spill-over of the exiting queue to the highway and no spillback to upstream intersection, (31) ensures that the queue length of each link cannot exceed a predefined threshold.

$$\sum_{i \in \psi(m)} (x^t_i - y^t_i) \leq L_m, \text{ for each link } m \tag{31}$$

D. Solution algorithm

As per the formulation, the model is a mix integer program and can be solved by general Integer Program algorithm such as Branch and Bound. But there will be 4 variables per cell pre time step and more than 10 variables per intersection per time step. So it can be expected that solving by the ordinary Integer Program Algorithm will be very slow when the simulation time is long. So in this study, the Genetic Algorithm (GA) is employed to find an almost optimal solution.

GA has been considered to be more and more promising for solving the real-world problems in the past decades. GA is a probabilistic search approach, which is founded on the ideas of evolutionary processes. The GA procedure is based on the Darwinian principle of survival of the fittest. An initial population is created containing a predefined number of individuals, each represented by a genetic string. Each individual has an associated fitness measure. The concept that the fittest (or best) individuals in a population will produce a fitter offspring is then implemented in order to reproduce the next population. Selected individuals are chosen for reproduction (or crossover) at each generation, with an appropriate mutation factor to randomly modify the genes of an individual, in order to develop a new population. The result is another set of individuals based on the original population leading to subsequent populations with better fitness and those with lower fitness will naturally get discarded from the population.

As in the Off-Ramp Integrated Control Model (ORICM), the decision variables are the signal timing settings. If the signal timing is set, the other parameters and the objective value can be computed by Cell Transmission procedure. So, in this study, a GA individual presents a set of signal plans. The GA procedure is illustrated in Fig. 6. The first population is generated randomly and each individual is decoded to a set of signal plan. Then, the Cell Transmission procedure will compute the objective value for one hour's simulation result. The corresponding fitness measure can be obtained from the objective function value. Based on the fitness evaluation, the crossover and mutation procedures are performed. This procedure will continue until the stop criterion is satisfied.

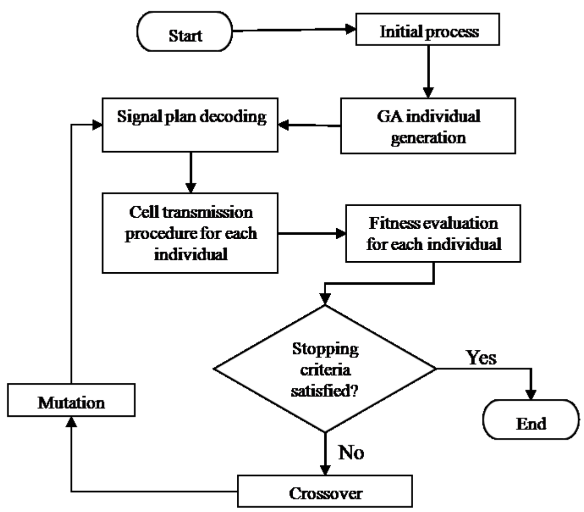

Fig. 6. Flowchart of GA

To accommodate the traffic signal optimization constraints, a fraction-based decoding scheme is developed based on the NEMA phase's structure as shown in Fig. 7.

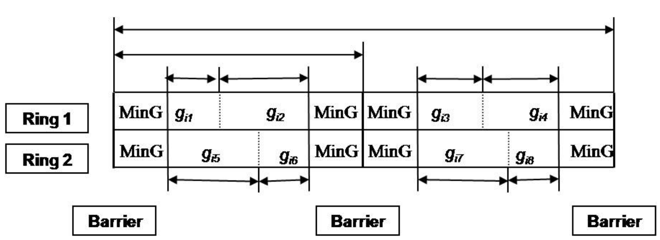

Fig. 7. Signal fraction-based decoding scheme

As illustrated by Fig. 7, six fractions are needed to describe the cycle length and split, and an additional one for offset. So, totally 7 fractions are needed for a four-leg NEMA coding signal scheme. Once the 7 fractions are generated, a signal plan can be decoded easily.

After the signal plans are generated, they are input to the cell transmission procedure, and the maximum queue length, total throughput and total delay are computed, which are used to evaluate the fitness of each signal plan.

III. Case Study

In this study, the I495@MD97 (see Fig. 2) interchange is considered to test the model. To compare the results, timing plans from Synchro and Off Ramp Integrated Control Model (ORICM) are input into CORSIM simulation package and results from one hour simulation are compared as follows.

For the entire control area, the total delay and off-ramp queue length are listed in Table 5. It is obvious that the maximum off-ramp volume which Synchro plan can accommodate is about 1400 vph. When the off-ramp volume is larger than this threshold, the exiting queue of the off-ramp will spillback to freeway. So, the conclusion for this case study is that for the presumed traffic pattern, when the off ramp volume is in the range 1400 to 2000 vph, the off-ramp integrated control strategy should be applied in terms of no spillback to freeway. The results also indicate that for off ramp volume ranging from 1400 to 1800 vph, the total delay experienced by the vehicles in the control area for Synchro plan is less than that of the ORICM, since in this volume range, a smaller number of vehicles enter the control area and the average delay is larger.

TABLE II
CONTROL AREA TOTAL DE LAY (veh-hr) COMPARISON

Off ramp volume (vphpl)	Synchro Model	ORICM	Synchro - ORICM
1000	134	136	-2
900	131	130	2
800	127	104	23
700	117	90	27
600	83	88	-5
500	80	80	0
400	70	75	-5

TABLE III
CONTROL AREA TOTAL THROUGHPUT (vph) COMPARISON

Off ramp volume (vphpl)	Synchro Model	ORICM	Synchro - ORICM
1000	4891	5377	-1387
900	4904	5459	-1702
800	4896	5314	-1061
700	4949	5124	-245
600	4908	4924	399
500	4718	4725	443
400	4521	4538	322

TABLE IV
CONTROL AREA AVERAGE DELAY (sec/veh) COMPARISON

Off ramp volume (vphpl)	Synchro Model	ORICM	Synchro - ORICM
1000	98	91	7
900	96	85	11
800	93	70	23
700	85	63	22
600	61	65	-4
500	61	61	0
400	56	59	-4

TABLE V
CONTROL AREA OFF-RAMP EXITING QUEUE LENGTH (veh) COMPARISON

Off ramp volume (vphpl)	Synchro Model	ORICM	Synchro - ORICM
1000	74	71	3
900	74	62	12
800	74	28	46
700	73	18	55
600	33	14	19
500	20	10	10
400	17	9	8

IV. CONCLUSIONS

In this study, a mix integer version of Integrated Off-Ramp Control Model and the GA based solution algorithm are proposed. To show the advantage of this model, the MD97@I495 interchange is employed as a numerical case study. The resulting plan is compared with the plan from Synchro and CORSIM simulation package. The comparison shows that the proposed model dominates Synchro plan in terms of total delay and average delay for certain range of off-ramp volume.

As shown above, to benefit the entire system, the effect of exiting off-ramp queue on upstream freeway traffic should be considered. In this study, to make things simple and to evade modeling this effect, the extra constraint is enforced to ensure that there is no exiting queue spillback to the freeway. This is a temporary solution as no exiting queue model is available.

There are two main issues in this model. First, the model does not include the freeway portion, i.e., optimization is confined to the arterial section. It is not necessarily optimal for the entire arterial and freeway system. There is a trade-off between the delay in arterial and the freeway. Second, the model cannot provide optimal results for over-saturated condition as effect of exiting queue on freeway cannot model by this model.

So a further research direction is to develop a model which considers the exiting queue's effect on freeway traffic. If that model is available, the proposed model can be modified to obtain system wide optimization.

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