# Optimal Control Strategies for Massive Vehicular-Pedestrian Mixed Flows

# in the Evacuation Zone

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# ABSTRACT

This paper presents an integrated model for design of signal plans for massive mixed pedestrian-vehicle flows within the evacuation zone. The proposed model with its embedded formulations for pedestrians and vehicles in the same network can effectively take into their potential conflicts during the evacuation, and generate the optimal routing strategies for guiding evacuees moving toward either the pick-up locations or their parking areas. The core formulations is based on the cell-transmission concept, but the proposed model has been enhanced with the notion of sub cells proposed mainly to capture the complex movements in the pedestrian flows so that it can concurrently optimize both the signals for pedestrian-vehicle flows and the movement directions for evacuees. An illustrating example concerning the evacuation around the M&T stadium area has been used to demonstrate the promising properties of our model.

# 1. INTRODUCTION

Due to potential terrorist attacks or other possible emergencies, evacuation related strategies and technologies have received increasing attention in recent years [1-4]. Liu et.al [5] has developed an evacuation system for Washington D.C., with an integrated optimization/simulation method. Nan, Cova and De Silva have proposed the application of simulation systems for computing the clearance time and identifying potential bottlenecks [6-11]. Most such evacuation studies consider mainly the passenger cars and their routing strategies, not the potentially large number of pedestrians who depend on transit or other modes for evacuation. In the process of accessing some designated pick-up points for evacuation, the massive pedestrian may incur tremendous burden to local signal control systems that are not designed to accommodate the large pedestrian flows. Thus, to prevent signalized intersections from becoming bottlenecks due to the massive mixed vehicle-pedestrian flows is one of the critical issues during evacuation operations, especially in metropolitan areas.

However, this vital issue of optimizing signals for vehicle-pedestrian flows has not been adequately addressed in the literature [12]. Some limited studies on this regard have attempted to use various modeling methods to capture pedestrian movements during evacuation [13-15]. For example, Massimo [16] uses a multimodal mesoscopic approach to model evacuation operations, where, pedestrians are guided through an emergency exit from buildings to a pick-up point by buses. However, most of such studies for pedestrian evacuation consider pedestrian's movement inside buildings or along the streets having no interactions with vehicle flows. This may not be in consistent with actual evacuation scenarios in most metropolitan areas, where pedestrian traffic with access only to public transit systems have to travel to the pick-up points to be rescued. Likewise, those having private vehicles also have to walk along the streets to their respective parking lots. Thus, there are some critical issues that need to be tackled by the responsible agencies. For instance:

1) How to guide pedestrians to their intended destinations when there are several available paths;

2) How to direct vehicles from the parking areas out of the evacuation zone under extremely congested vehicle-pedestrian flows; and

3) How to coordinate the vehicle and the pedestrian flows through intersections and oversaturated local streets.

For example, if an evacuation operation for the M&T football stadium in Baltimore downtown area is taking place (see Figure 1), the pedestrian will have to cross the streets to get to their parking lots or transit stops. Consequently some conflicts between pedestrians and vehicle flows may incur in the evacuation zones. To minimize such potential conflicts, one firstly needs to identify the possible paths between each pedestrian or vehicle O-D pair, and then provide the guidance for pedestrians or vehicles to distribute among the possible paths. Since there are a large number of pedestrians to be evacuated, it poses a challenge to responsible agencies on how to coordinate these massive pedestrian and vehicle flows via effective signal control strategies. In fact, failing to account for the conflicts between these two flow movements in the signal design may result in the following operation issues:

1) Insufficient green time split for pedestrian flows will cause long pedestrian queues and long waiting times for evacuees, and may thus incur chaotic in the evacuation zones. This is due partly to the fact that some in the pedestrian flows are likely to be passenger-car drivers or riders. The blockage of

pedestrian flows may therefore hinder the vehicle flows ready to depart from the parking areas.

2) In contrast, insufficient green time for vehicle flows will certainly cause long queues and spillback to the parking area, which will prevent evacuees from leaving the evacuation zones efficiently even if they are able to access their parking garages.



FIGURE 1 A graphic illustration of the intersection of Hamburg Street@MD295 near M&T stadium.

Hence, an integrated model to coordinate both the pedestrian and vehicle flows during evacuation operations is certainly an imperative task. Such an operational model for guiding and control vehicle-pedestrian flows shall have the following key features:

1) Realistically represent the networks of vehicle and pedestrian flows, and capture their interactions;

2) Integrate these two networks and compute the optimal vehicle departure rate for each intersection approach based on the pedestrian arrival rate;

3) Generate the signal phase plan that can fully account for both massive pedestrian and vehicle flows; and

4) Concurrently optimize the routing strategy and signal timings for both pedestrians and vehicle flows within their conflicting zones.

This paper intends to address our research on the aforementioned issues, and is organized as follows: Firstly, the formulations with the enhanced cell-transmission concept [17][18] for the integrated network that contains both the vehicle and the pedestrian flows will be presented in the next section. Secondly, a signal optimization model that account for both vehicle and pedestrian flows as well as their routing strategies within the evacuation zones will be presented in Section 3. Thirdly the solution algorithm developed with the GA algorithm is illustrated in Section 4. Lastly, an illustrative example with M&T stadium evacuation is presented in the last section along with the conclusions.

# 2. CELL REPRESENTATION OF THE INTEGRATED NETWORK

#### 2.1 Cell Representation of the Vehicle Network

Cell transmission model was originally proposed by Daganzo [17] [18] to simulate traffic flow based on hydrodynamic traffic model introduced by Lighthill and Whitham [19] and Richards [20]. Its core concept is to decompose the transportation network into cells. The length of each cell is the maximum distance that a vehicle can travel in a unit time. Since the CTM by Daganzo doesn't explicitly model signalized intersection, Ziliaskopoulos and Lee [21] designated a single cell for each turning movement. Here, we adopt this model to build the vehicle network and simulate the vehicle flow dynamics. To facilitate the description, we introduce the CTM model in Figure 2:



 $y_{i,j,t} = \min(Q_{i,t}, Q_{j,t}, n_{i,t}, N_{j,t} - n_{j,t})$ 

FIGURE 2 A graphic illustration of the CTM model.

where  $y_{i,j,t}$  is the flow from cell i to cell j at time t;  $Q_{i,t}$  is the saturation flow the ith cell can receive or push at time t;  $n_{i,t}$  is the current number of vehicles in cell i at time t; and  $N_{i,t}$  is the maximum number of vehicles cell i can hold at time t.

#### 2.2 Cell Representation of the Pedestrian Network

In general, all movements during evacuation may take place in one of the following areas: inside-building area, open area, sidewalks, and intersection crossings. Since the focus is on guiding and controlling pedestrian-vehicle flows within the evacuation zones, this study will present our modeling efforts on the interactions of the pedestrian-vehicle flows over sidewalk and at intersection crossing areas. Movements in all other parts are modeled either as source or sink nodes where pedestrians can be loaded in or out of the pedestrian network. Figure 3 presents the walking facilities at the intersections around the M&T stadium (see the left part), and its conversion representation as a cell transmission network (see the right part).



FIGURE 3 Walking facilities and their cell representations.

The primary differences between the pedestrian and vehicle flows are that: (1) pedestrians can move toward their preferred directions at any point. For instance, pedestrian can move in bi-directional way in a sidewalk and crossing zones, thus impacting the saturation flow of each direction; (2) each pedestrian cell may have multiple upstream cells and downstream cells; and (3) conflicts in the pedestrian flows may incur frequently when evacuees are moving toward different directions. Some critical issues associated with the modeling of pedestrian network are presented below.

#### 2.2.1 Determine the Saturation Flow Rates for Each Direction

Assuming that there are two pedestrian groups moving toward different directions in a cell C, one can then split the cell into two sub cells  $C_f$  and  $C_b$ , each representing the pedestrian group moving in a certain direction as depicted in Figure 4 below:



FIGURE 4 Sub cells representation of pedestrian groups.

Let  $Q_{C_f,t}$  and  $Q_{C_b,t}$  respectively, be the saturation flows for sub cells  $C_f$  and  $C_b$  that can receive or push at time t;  $Q_{C,t}$  be the saturation flow the cell C can receive and push at time t;  $n_{C_f,t}$  be the number of pedestrians in the cell  $C_f$  at time t;  $n_{C_b,t}$  be the number of pedestrians in the cell  $C_b$  at time t. One can then establish the following relationships:

$$Q_{C_{f},t} + Q_{C_{b},t} = (1 - \alpha)Q_{i,t}$$
(1)

$$\frac{Q_{C_{b,t}}}{Q_{C_{c,t}}} = f(n_{C_{b,t}}, n_{C_{f,t}})$$
(2)

$$\alpha = g(n_{C_b,t}, n_{C_f,t}) \tag{3}$$

Equation (1) implies that the sum of the saturation flow for both sub cells should be less than or equal to the total saturation flow of the cell, especially under over-saturated condition, where frictions between opposite pedestrian flows will reduce the saturation flow. Let  $\alpha$  be the friction coefficient, and it shall be a function of pedestrians in each sub-cell. Equation (2) shows that the saturation flow for each direction is affected by the numbers of pedestrians in both directions, usually the more pedestrians present in a sub cell, the more space the sub cell occupies, and thus the more saturation flow the sub cell has. For convenience of model presentation, this study takes into the following simple forms for these two parameters.

$$\alpha = c \tag{4}$$

$$f(n_{C_{b,t}}, n_{C_{f,t}}) = \frac{n_{C_{b,t}}}{n_{C_{f,t}}}$$
(5)

Equation (4) implies that the saturation flow reduction coefficient is a constant, independent of the number in each sub-cell. Equation (5) illustrates that the saturation flow for one sub cell is proportional to the number of pedestrians present in that cell. This is consistent with the fact that the larger pedestrian group often occupy more space, and thus generate larger traffic flows.

#### 2.2.2 Determining the Flow for a Generalized Cell

Consider a sub cell  $C_{i,b}$  in pedestrian cell  $C_i$ , and it has *m* downstream cells, numbered from i + 1

to i + m, shown in Figure 5:



FIGURE 5 Sub-cell connecting downstream cells.

The variables are defined as:

 $P_{i,j,t}$ : The flow that can be sent from cell *i* to cell *j* at time *t*;

 $r_{i,j,t}$ : The ratio of pedestrians in the sub-cell,  $C_{i,b}$ , intending to move from cell *i* to cell *j* at time *t*.

One can have the following relationship:

$$\sum_{j=i+1}^{i+m} P_{i,j,t} = \min(Q_{C_{i,b},t}, n_{C_{i,b},t})$$
(6)

Equation (6) implies the sum of flows that sub cell  $C_{i,b}$  can send out can't exceed the saturation flow from sub-cell  $C_{i,b}$ , and the total number in sub-cell,  $C_{i,b}$ .

To determine the value for each  $P_{i,j,t}$ , one can further divide the first-level sub cell  $C_{i,b}$  to n secondary-level sub cells labeled from  $C_{i,b,1}$  to  $C_{i,b,n}$ , each representing a pedestrian group that will move to a certain downstream cell, as depicted in figure 6:



FIGURE 6 Secondary sub-cells.

Let  $r_{i,j,t}$  be the turning ratio in the first-level sub cell *i* that will move to cell *j*, we can establish the following relationships:

$$\frac{Q_{C_{i,b,k},t}}{Q_{C_{i,b,l},t}} = f_k(n_{C_{i,b,1},t}, n_{C_{i,b,2},t}, \dots, n_{C_{i,b,m},t})$$
(7)

$$n_{C_{i,b,j},t} = r_{i,i+j,t} n_{C_{i,b},t}$$
(8)

$$P_{i,i+j,t} = \min(Q_{C_{i,b,j},t}, n_{C_{i,b,j},t})$$
(9)

Equation (7) shows that the saturation flows of every second-level sub-cell interact with each other, depending on their accommodated numbers of pedestrians. Equation (8) reflects that the number in each second-level sub cell is proportional to the corresponding turning ratio in the first-level sub-cell. Equation (9) implies that the maximal outgoing flow from cell i to cell j at time t is constrained by the saturation flow of the second-level sub cell and the number intending to move from cell i to cell j at time t.

Assuming that the saturation flow of each secondary level sub cell at time t is proportional to its number of pedestrians in that cell at time t, we can represent the function in the following form:

$$f_k(n_{C_{i,b,1},t}, n_{C_{i,b,2},t}, \dots, n_{C_{i,b,m},t}) = r_{i,i+k,t}$$
(10)

In summary, one can represent the maximal outgoing flow from cell *i* to cell *j* at time t as follows:

$$P_{i,j,t} = \begin{cases} \min(r_{i,j,t} n_{C_{i,b},t}, \frac{(1-\alpha)r_{i,j,t} n_{C_{i,b},t} Q_{i,t}}{n_{C_{i,b},t} + n_{C_{i,f},t}}), if(n_{C_{i,f},t} > 0) \\ \min(r_{i,j,t} n_{C_{i,b},t}, r_{i,j,t} Q_{i,t}), if(n_{C_{i,f},t} = 0) \end{cases}$$
(11)

For a pedestrian cell *i* having *m* upstream cells, one can number these from i + 1 to i + m below:



FIGURE 7 Pedestrian cell with multiple upstream links.

Let  $S_{i,t}$  be the maximum inflow to cell *i* at time *t*, and then it exists the relationship below:

$$S_{i,t} = \begin{cases} \min((1-\alpha)Q_{i,t}, N_{i,t} - n_{i,t}), if(n_{i_{-1},t} > 0, n_{i_{-2},t} > 0) \\ \min(Q_{i,t}, N_{i,t} - n_{i,t}), else \end{cases}$$
(12)

Equation (12) shows that the maximum inflow is constrained by the maximal saturation flow of the cell and the remaining space in that cell.

Let  $y_{i,j,t}$  be the pedestrian flow moving from cell *i* to cell *j* at time *t*, one can then establish the following equation:

$$y_{i,j,t} = \begin{cases} P_{i,j,t}, if(\sum_{s} P_{s,j,t} \le S_{j,t}) \\ \min(P_{i,j,t}, S_{j,t}g_{i,j,t}), if(\sum_{s} P_{s,j,t} > S_{j,t}) \end{cases}$$
(13)

Equation (13) shows that if a pedestrian cell has enough space to accommodate the flows from all its upstream pedestrian cells, the flow from pedestrian cell i to cell j is constrained by the maximum outgoing flow from pedestrian cell i to pedestrian cell j. Otherwise, the flow from pedestrian cell i to pedestrian cell j constrained by the total pedestrians that the pedestrian cell j can accommodate.

Again, for convenience of presentation, let the fraction coefficient of the movement from pedestrian cell i to cell j be defined as proportional to the maximum pedestrian flow that can be sent by pedestrian cell i to pedestrian cell j, and be expressed as follows:

$$g_{i,j,t} = \frac{P_{i,j,t}}{\sum_{i} P_{i,j,t}}$$
(14)

Thus,

$$y_{i,j,t} = \begin{cases} P_{i,j,t}, if(\sum_{s} P_{s,j,t} \le S_{j,t}) \\ S_{j,t} \frac{P_{i,j,t}}{\sum_{s} P_{s,j,t}}, if(\sum_{s} P_{s,j,t} > S_{j,t}) \end{cases}$$
(15)

#### 2.2.3 Determine Flow for Conflict Movements

In some case, there exist conflict flows in the pedestrian network, for example, at un-signalized intersections, as shown in Figure 8:



FIGURE 8 Conflicting pedestrian movements.

Let the flows for each direction be denoted as  $q_i$ ,  $i \in (1, 2, ..., n)$ , and its saturation flow be

defined as  $Q_i$ ,  $i \in (1, 2, ..., n)$  which is the maximal number of pedestrians that can pass along their target directions without incurring any interference with those moving in other directions. Then, the following relationship shall exist:

$$\sum_{i} \frac{q_i}{Q_i} \le 1 \tag{17}$$

Note that Equation (17) shows the constraint on the conflict flows. Assuming that one can divide the time into small slices and during each only the pedestrians from one direction can pass the conflicting

point, then  $\frac{q_i}{Q_i}$  is the percentage of the total time the flow of direction *i* occupies the conflict point. If the

sum of these percentages is equal to 1, it implies that the conflict point keeps occupied over the target time duration

#### 2.3 Connecting the Pedestrian Network and the Vehicle Network

For the pedestrians without access to individual vehicles, their destinations are the pick-up points where buses will load them to safe areas. For others, their destinations are the parking lots where passenger cars are loaded onto the vehicle network. To realistically capture the interactions of mixed flow movements, the pedestrian network and the vehicle network should be connected. Here we define the connection cell to be a kind of cells connecting the two networks, which is also the sink cell of the

pedestrian network and the source cell of the vehicle network. The connection cell, as depicted in Figure 9, denote the locations where the pedestrians will get onto the vehicles, for example, parking lots and transit stops:



FIGURE 9 Connection cell.

For each connection cell, one can define  $\tau$  to be the average individual delay incurred in accessing his/her car. One can also define  $\lambda$  to be the mean carpooling rate, and then the following relationships hold:

$$n_{i,t}^{ped} = \sum_{t_0=t-\tau}^{t-1} y_{i-1,i,t_0}$$
(18)

$$n_{i,t}^{veh} = n_{i,t-1}^{veh} + \frac{y_{i-1,i,t-\tau}}{\lambda} - y_{i,i+1,t-1}$$
(19)

Equation (18) updates the number of pedestrians in the connection cell at time t. It is the number of pedestrians entering the connection cell within the time window  $(t - \tau \text{ and } t)$ . Equation 19 updates the

number of vehicles preparing to enter the vehicle network. The term,  $\frac{y_{i-1,i,t-\tau}}{\lambda}$  is the number of vehicles

generated by the pedestrian flows entering the network at time  $t - \tau$ , and the  $y_{i,i+1,t-1}$  is the number of vehicles flows moving into the vehicle network at time t - 1

# 3. SIGNAL OPTIMIZATION

# 3.1 Signal Phase Design for Intersections with Pedestrian Crosswalks

A ring-and-barrier diagram is selected to represent the phase sequence of a signal controller. Taking a typical four-leg intersection for example, the normal signal phase sequence is depicted in Figure 10.



FIGURE 10 Ring-and-barrier diagram for a typical four-leg intersection.

Note that under the typical design, the right-turn vehicles should always yield to the pedestrian traffic. However, during evacuation the pedestrian-crossing flow is always over-saturated, which may

incur the blockage of the right-turn traffic. To address this critical issue, we propose an exclusive phase to the right-turn traffic, as shown in the Figure 11:



FIGURE 11 Ring-and-barrier diagram with an additional exclusive right turn phase.

# **3.2 Signal and Routing Optimization Formulations**

The model formulation for combined signal and routing optimization are mostly divided into the following sub-problems with the same objective, i.e., maximize the total system throughput from the evacuation zone [22 - 27], and solve them iteratively to get the combined result:

- 1) optimize the routing strategy based on the fixed signal settings; or
- 2) optimize the signal settings for a fixed set of routing strategies.

A system-optimal formulation based on the cell transmission was proposed by Christopher and Ziliaskopoulos [28]. We have extended their formulations below to accommodate the pedestrian flows.

The decision variables are defined as:

 $n_{o,d,i,j,t}$ : The number of pedestrians (vehicles) in cell *i* to cell *j* with source cell *o* and sink cell *d* at time *t*;

 $y_{o,d,i,j,t}$ : The number of pedestrians (vehicles) from cell *i* to cell *j* with source cell *o* and sink

cell d at time t;

 $W_{i,i,k,t}$ : Indicator to show whether the light is green at intersection k from cell *i* to cell *j* at time *t*.

All other variables are defined as:

**TABLE 1 Variable Definition** 

Variable	Definition
λ	The average carpooling rate
С	Set of cells, pedestrian $C^p$ , vehicle $C^v$ , pedestrian source $C_r^p$ , pedestrian sink $C_s^p$ , vehicle source $C_r^v$ , vehicle sink $C_s^v$
$P_{o,d,i,j,t}$	The number of pedestrians (vehicles) that can be sent from cell $i$ to cell $j$ with source cell $o$ and sink cell $d$ at time $t$
$Z_{i,j,t}$	Indicator to show whether the movement from cell $i$ to cell $j$ at time $t$ is allowed
М	An extra large number
$\mathcal{Q}_{i,t}$	The saturation flow rate of cell <i>i</i> at time <i>t</i>
S <sub>i,t</sub>	The number of pedestrians (vehicles) cell <i>i</i> can receive at time <i>t</i>
$N_{i,t}$	The maximum number of pedestrians (vehicles) cell $i$ can hold at time $t$
$r_{i,1,t}$	Indicator whether ring 1 of intersection <i>i</i> is active at time <i>t</i>
$r_{i,2,t}$	Indicator whether ring 2 of intersection $i$ is active at time $t$
G <sub>max</sub>	The maximum green time
$G_{\min}$	The minimum green time
τ	The delay between exit and entry in the connection cell
Ι	Set of intersections
$A_k$	Set of movement of intersection k
$H_k$	Set of conflict movement of intersection $k$
B <sub>1</sub>	The $l$ th set of conflict pedestrian movements without signal controller
$D_{i,1,t}$	Set of downstream cells for the first movement direction in cell $i$ at time $t$
$D_{i,2,t}$	Set of downstream cells for the first movement direction in cell <i>i</i> at time <i>t</i>

We then present the objective function and the all assorted constraints below:

Maximize 
$$\sum_{\forall d \in C_s^p} n_{o,d,d,T} + \lambda \sum_{\forall d \in C_s^v} n_{o,d,d,T}$$

Subject to

$$P_{o,d,i,j,t} \le n_{o,d,i,j,t} \qquad \forall o \in \{C_r^v, C_r^p\}, \forall d \in \{C_s^v, C_s^p\}, \forall i, j \in \{C^v, C^p\}, \forall t \in T$$
(20)

$$P_{o,d,i,j,t} \le z_{i,j,t} * M \qquad \forall o \in \{C_r^v, C_r^p\}, \forall d \in \{C_s^v, C_s^p\}, \forall i, j \in \{C^v, C^p\}, \forall t \in T$$

$$\tag{21}$$

$$\sum_{o} \sum_{d} \sum_{j} P_{o,d,i,j,t} \leq Q_{i,t} \quad \forall i \in C^{\nu}, \forall t \in T$$

$$(22)$$

$$P_{o,d,i,j,t} \leq \begin{cases} \frac{n_{o,d,i,j,t}}{\sum_{o} \sum_{d} \sum_{r} n_{o,d,i,r,t}} Q_{i,t}, if(\sum_{j \in D_{i,1,t}} \sum_{o} \sum_{d} n_{o,d,i,j,t} = 0OR \sum_{j \in D_{i,2,t}} \sum_{o} \sum_{d} n_{o,d,i,j,t} = 0) \\ \frac{(1-\alpha)n_{o,d,i,j,t}}{\sum_{o} \sum_{d} \sum_{r} n_{o,d,i,r,t}} Q_{i,t}, else \end{cases}$$

 $\forall i, j \in C^p, \forall t \in T$ 

 $S_{i,t} \le Q_{i,t} \qquad \forall i \in C^{\nu}, \forall t \in T$ (24)

$$S_{i,t} \leq \begin{cases} Q_{i,t}, if(\sum_{j \in D_{i,1,t}} \sum_{o} \sum_{d} n_{o,d,i,j,t} = 0 OR \sum_{j \in D_{i,2,t}} \sum_{o} \sum_{d} n_{o,d,i,j,t} = 0) \\ (1 - \alpha)Q_{i,t}, else \end{cases} \quad \forall i \in C^{p}, \forall t \in T$$

$$(25)$$

$$S_{i,t} \le N_{i,t} - \sum_{o} \sum_{d} \sum_{j} n_{o,d,i,j,t} \qquad \forall i \in \{C^{\nu}, C^{p}\}, \forall t \in T$$

$$(26)$$

$$y_{o,d,i,j,t} \le P_{o,d,i,j,t} \qquad \forall o \in \{C_r^v, C_r^p\}, \forall d \in \{C_s^v, C_s^p\}, \forall i, j \in \{C^v, C^p\}, \forall t \in T$$

$$(27)$$

$$\sum_{o} \sum_{d} \sum_{i} y_{o,d,i,j,t} \leq S_{j,t} \quad \forall j \in \{C^{\nu}, C^{p}\}, \forall t \in T$$

$$(28)$$

$$\sum_{o} \sum_{d} \sum_{j} n_{o,d,i,j,t} = \sum_{o} \sum_{d} \sum_{j} n_{o,d,i,j,t-1} - \sum_{o} \sum_{d} \sum_{j} y_{o,d,i,j,t-1} + \sum_{o} \sum_{d} \sum_{k} y_{o,d,j,i,t-1}$$
$$\forall i \in \{C^{v} \setminus \{C_{r}^{v}\}, C^{p} \setminus \{C_{s}^{p}\}\}, \forall t \in T$$
(29)

$$\sum_{o} \sum_{d} \sum_{j} n_{o,d,i,j,t} = \sum_{o} \sum_{d} \sum_{j} n_{o,d,i,j,t-1} - \sum_{o} \sum_{d} \sum_{j} y_{o,d,i,j,t-1} + \frac{\sum_{o} \sum_{d} \sum_{k} y_{o,d,j,i,t-\tau}}{\lambda}$$
$$\forall i \in \{C_{r}^{v}\}, \forall t \in T$$
(30)

(23)

$$\sum_{(i,j)} \frac{y_{o,d,i,j,k}}{S_{j,t} - \sum_{k \neq i}} \sum_{o} \sum_{d} y_{o,d,k,j,t} \le 1, \quad \forall o \in C_r^p, \forall d \in C_s^p, \forall l, \forall (i,j) \in B_l$$
(31)

$$r_{i,1,t} + r_{i,2,t} = 1 \qquad \forall i \in I, \forall t \in T$$
(32)

$$\sum_{(i,j)\in R_k^1} w_{i,j,k,t} = 2r_{k,1,t} \ \forall k \in I, \forall t \in T, \forall (i,j) \in A_k$$
(33)

$$\sum_{(i,j)\in R_k^2} w_{i,j,k,t} = 2r_{k,2,t} \qquad \forall k \in I, \forall t \in T, \forall (i,j) \in A_k$$
(34)

$$W_{i,j,k,t} + W_{i',j',k,t} \le 1 \qquad \forall k \in I, \forall (i,j,i',j') \in H_k$$
(35)

$$\sum_{\tau=t}^{t+G_{\max}+1} W_{i,j,k,\tau} \le G_{\max} \qquad \forall k \in I, \forall (i,j) \in A_k$$
(36)

$$\sum_{\tau=t}^{t+G_{\min}} w_{i,j,k,\tau} \ge (w_{i,j,k,t} - w_{i,j,k,t-1})G_{\min} \quad \forall k \in I, \forall (i,j) \in A_k$$

$$(37)$$

$$z_{i,j,t} = \begin{cases} 1, if(w_{i,j,k,t} = 1)Or(\sum_{\forall (l,m,l,j) \in H_k} \sum_o \sum_d \sum_{\tau=t}^{l+g} n_{o,d,l,m,\tau} = 0) \\ 0, otherwise \end{cases} \quad \forall k \in I, \forall (i,j) \in A_k \tag{38}$$

$$y_{o,d,i,j,0} = 0$$
 (39)

$$y_{o,d,i,j,0} \ge 0 \tag{40}$$

$$n_{o,d,i,j,t} \ge 0 \tag{41}$$

The objective function is to maximize the total throughput at time T in the unit of person, including those with and without access to passenger cars

Constraints (20) through (30) extend the original model to capture the flow dynamics and their interactions between pedestrians and vehicles. Constraint (23) and constraint (25) distinguish different saturation flow conditions under one-directional or bi-directional flow movement in a certain pedestrian cell. Constraint (30) pertains to the number update in the connection cells. Constraint (31) deals with conflict pedestrian flows without signal coordination. Constraints (32) to (38) are to set the restrictions on the traffic signal timings, and constraints (39) to (42) are used to initialize the variables.

# **3.3 Genetic Algorithm for Solving the Problem**

Foy et al [29] were the first to apply GA for signal timing computation, Hadi and Wallace [30] combined GA with TRANSYT-7F to optimize signal phase sequence, cycle length, green splits, and offset. There are other related literatures [31- 38], where most reported that GA-based algorithms perform better

at generating timing plans than most optimization tools, despite its computational intense nature.

Note there are three kinds of decision variables in the proposed formulations:  $n_{o,d,i,j,t}$ ,  $y_{o,d,i,j,t}$ and  $w_{i,j,k,t}$ . However, these variables are tightly constrained, so it is not advisable to directly encode them to GA code. Rather, we reselect and rewrite the variables in the following forms:

$$n_{o,d,i,j,t} = \gamma_{o,d,i,j,t} n_{o,d,i,t}$$
(43)

$$\sum_{j} \gamma_{o,d,i,j,t} = 1 \tag{44}$$

$$y_{o,d,i,j,t} = \min(P_{o,d,i,j,t}, r_{o,d,i,j,t} * (S_{j,t} - \sum_{\forall \{o',d',i'\} \neq \{o,d,i\}} y_{o',d',i',j,t}))$$
(45)

$$\phi_{i,j} = \Phi_{i,\min} + \rho_{i,j} \left( \Phi_{i,\max} - \Phi_{i,\min} \right)$$
(46)

In equation (43), the pedestrians in a cell are divided into several groups by using turning, each moving to a certain downstream cell. Equation (44) put a constraint on the turning ratios to make them feasible. In equation (45), the flow from cell *i* to cell *j* is constrained by the flow that cell *i* can send out and the remaining space the cell *j* can still accommodate. In equation (46), each green time of the signal phase lie between its minimum and maximum green times;  $\phi_{i,j}$  is the green time of the *i* th phase at the *k* th cycle;  $\Phi_{i,\text{max}}$  is the maximum green time for the *i* th phase, and  $\Phi_{i,\text{min}}$  is the minimum green time for the

i th phase.

The  $\gamma_{o,d,i,j,t}$ ,  $r_{o,d,i,j,t}$  and  $\rho_{i,i}$  composes the GA code, they are within the range [0, 1]. The

standard procedure of a uniform cross-over [39] is applied here. Two parents are combined to produce two new offsprings. Individual bits in the string are compared between two parents. The bits are swapped with a fixed probability, typically 0.5. A mutation occurs in a certain probability to each bit of the string. An example of the uniform cross-over and mutation operation is depicted in Figure 12



FIGURE 12 Illustrative example of uniform cross-over and mutation.

# 4. ILLUSTRATIVE EXAMPLE

This section presents an illustrative case with the proposed model, using the M&T stadium in Baltimore downtown area. It is assumed to have 20000 individuals who need to evacuate from the stadium.



The satellite image and the parking lot layout around the M&T stadium are depicted in Figure 13:

Figure 13 Layout of the M&T stadium in Baltimore city.

The layout of the pedestrian cell network is depicted in Figure 14. It is assumed that pedestrians can move from one cell to any of its adjacent cells. The cells 1 to 49 are ordinary pedestrian cells. The cell 1000 is the pedestrian source cell; the sink cells 101 to 105 are parking lots; and the sink cell 106 is the pick-up point for those without access to passenger cars.



FIGURE 14 Cell representation of the pedestrian network.

The demand of each pedestrian O-D pair is listed in the table below, and it is estimated based on the capacity of the parking lots:

Destination	Estimated O-D demand	Estimated Parking Capacity	
	(# of individuals)	(vehicles)	
101	6800	4000	
102	1700	1000	
103	3400	2000	
104	850	500	
105	4250	2500	
106	3000	Transit stop	

**TABLE 2 O-D Pair for Pedestrians** 

The layout of the vehicle network is illustrated in Figure 15. The arrows indicate the possible flow directions between cells. The cells 201 to 235 are ordinary vehicle cells. The cells 101 to 105 are vehicle source cells, which are also the pedestrian sink cells. The sink cells 301, 302, 303, 304 and 305 can be viewed as destinations for vehicles intended to get onto I83, US40, I395 South, MD295 South, and MD2, respectively.



FIGURE 15 Cell representation of the vehicle network.

The demand of each O-D pair is listed in the table below, and it is based primarily on zip codes of all seasoned ticket holders [40]:

0 D	301	302	303	304	305
101	653	481	505	1761	0
102	95	0	0	755	0
103	0	830	870	0	0
104	0	207	218	0	0
105	0	950	996	0	179

 TABLE 3 O-D Pair for Passenger Cars (Unit: # of Vehicles)

Four signal controllers and one pair of conflict flows are present in our example case. To reduce the computation time, the phase diagrams are simplified to accommodate the restricted turning movements. It is assumed that some particular movements may not be allowed during the evacuation so as to efficiently control the evolution traffic flows. The allowable movements of each phase by all four signal controllers are listed in the Table 4:

TABLE 4 Cell Movements for Each Phase at the Intersection Hamburg Street@MD295

Controller Phase	C1	C2	C3	C4
$\phi_1$	217-221 219-221	204-201 205-208	223-228 229-228	227-226
$\phi_2$	212-221 2-9 3-9 16-9	209-201	35-37 36-38 39-38	42-43
$\phi_3$	216-213 218-214			
$\phi_4$	5-4 10-4 2-3 9-3			
$\phi_5$	222-214 4-10 5-10 12-10 17-10			

Take controller 1, for example, the ring-and-barrier diagram for the phases are depicted in Figure 16:



# FIGURE 16 Signal Phase at the intersection Hamburg street@MD295.

After applying the proposed model and solution algorithm to the test network, the resulting optimal plan includes

- 1) The signal timing for each phase
- 2) Pedestrian routing for each source node to each destination node
- 3) Vehicle routing plan

Table 5 lists the optimal timing information for all phases over consecutive cycles. It is assumed that the maximum green time is 100s per phase, the minimum green time is 10s, the yellow time between phases is 2s; and the maximum cycle length is 200s. The first cycle will begin when the first pedestrian or vehicle needs to cross the street:

	Phase 1 (s)	Phase 2 (s)	Phase 3 (s)	Phase 4 (s)	Phase 5 (s)
Cycle 1	95	90	78	15	90
Cycle 2	97	82	77	18	82
Cycle 10	92	78	61	29	78
Cycle 11	95	81	59	34	81
Cycle 20	87	76	32	53	76

TABLE 5 Optimized Signal Timings at Intersection Hamburg Street@MD295

Table 6 lists the possible paths of pedestrians and for each O-D pair, the demand of each path, and their corresponding clearance time (the time when all demands on that path have been cleared):

O-D	Used path	Path demand	Clearance
		(# of individuals)	time (min)
1000-101	1000-18-11-101	4550	84
	1000-22-20-17-12-13-14-15-11-101	1146	78
	1000-22-20-17-10-5-6-101	1104	70
1000-102	1000-22-20-17-10-4-3-2-1-102	1700	85
1000-103	1000-32-34-36-38-37-103	2126	58
	1000-42-41-40-39-38-37-103	1274	61
1000-104	1000-42-43-104	850	32
1000-105	1000-18-44-45-46-47-48-49-105	4250	66
1000-106	1000-22-20-17-10-4-106	1000	74
	1000-18-11-8-7-6-5-4-106	2000	67

**TABLE 6 Possible Paths for Pedestrian OD** 

Figure 17 shows the throughput of the 6 destinations (1 for transit stop, 5 for vehicles) with time, and the total evacuation clearance time is estimated to be around 1 hour and 40 minutes.



FIGURE 17 Throughput for various destinations.

# 5. SUMMARY AND CONCLUSION

This paper has presented an integrated control model to optimize the pedestrian and vehicle movements within the evacuation zone. The proposed model employs the cell-transmission concept to represent the pedestrian and vehicle networks, and formulate the interactions of these two types of massive flows at intersections. Based on the locations of pick-up point for evacuees using transit systems, and parking garage distributions for these having access to vehicles, the proposed model is capable of producing the routing strategies to guide pedestrians, responsive cycle length, and signal timing at each intersection within the evacuation zone.

An illustrative example presented in the paper seems to indicate that the pioneering study offers some promising properties to address the complex interactions between vehicle and pedestrian flows

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within the evacuation zone. Note that this work is exploratory in nature, and we fully recognize that much remains to be done in developing an efficient and operational evacuation system that can guide evacuees to the most proper mode and direct various types of traffic flows to the most efficient routes. Our ongoing research along this line is to develop an optimal location model for identifying the bus pick-up points, based on the time-varying demand, and then design the routing strategies to concurrently move both bus and passenger-car flows within a specified safety window.

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