A CONTINUUM MODELING APPROACH FOR ROAD PRICING FEASIBILITY STUDY:
A CASE STUDY AND SENSITIVITY ANALYSIS FOR BALTIMORE METROPOLITAN REGION

ABSTRACT

Road pricing has widely been considered by system planners for relieving congestion and generating revenue. The design and evaluation of different road pricing schemes are always necessary for the decision makers to ensure the potential beneficiary of road pricing and the practical implementation of the scheme. This study presents an evaluation procedure based on the use of continuum modeling approach for addressing the road pricing feasibility problem in Baltimore Metropolitan Region. In this study, Baltimore City is considered as the central business district (CBD) and the corresponding user-equilibrium, system-optimal and cordon-based charging models are setup and solved. Sensitivity analyses of total demand, total toll collected, social benefit and the location/shape of the charging cordons for this proposed model with respect to the input parameters are also completed. The study demonstrates the potential applicability of using continuum modeling approach for road pricing feasibility study in Baltimore Metropolitan Region.

1. Introduction

With a total of 56.8 million hours of travel delay and a wastage 40.8 million gallons of fuel in 2005 (Texas Transportation Institute, 2007), urban area of Baltimore is one of the most heavily congested areas in United States. Also, over the pass 20 years, the annual delay experienced by each peak hour traveler has increased by three times, from 11 hours to 44 hours, (Texas Transportation Institute, 2007). Although, Baltimore have adopted various operational treatments like freeway incident management, arterial street signal coordination and arterial street access management to alleviate congestion, these measures could not offset the congestion caused by the rapidly increasing demand of traffic. Among the ways to relief congestion, road pricing is the most straightforward approach as road users are directly charged on how much delay they have imposed on the system (marginal cost pricing). The most well-known and successful implementations of road pricing are in Singapore and London. In Singapore, after 3 months of the implementation of Area Licensing Scheme (ALS), the number of vehicles entering the charging zone is reduced by 45% (Phang and Rex, 2004). For London, after 1 year of implementation of congestion charging, the delay within the charging zone is reduced by 30% (Transport for London, 2004). With successful experience in Singapore and London, road pricing seems to be a promising way for reducing congestion and generate revenue in Baltimore.

For the implementation of road pricing scheme, one of the most critical issues to resolve is the toll collection method. Unlike collecting toll for tolled facility (e.g. bridge), the using of toll booths and toll plaza is not possible. It is because using these facilities will normally reduce the speed of vehicles and therefore causing delay and congestion in those areas. Also, the charging points/cordons of road pricing scheme is normally located within the developed and highly dense urban area, therefore it is usually no room for building toll plaza. Thus, in order to have a successful implementation of road pricing, an efficient way to collect toll without slowing the traffic should be establish. E-ZPass (E-ZPass, 2008), which are commonly used on Maryland (45% of total traffic in Maryland, (MdTA, 2005)) and the northeastern of US, provided an effective mean of toll collection. Currently, although toll booths are still needed for using the E-ZPass in Maryland, the idea of open road tolling, which tolls are collected electronically at highway speeds without the need for toll booth, in using E-ZPass have been introduced in Illinois states tollway (Illinois Tollway, 2008) and is also planned to be included in the I-95 Express Toll Lane (MdTA, 2005). With the introduction of open road tolling and the increasing usage of E-ZPass, road pricing is technically possible for the implementation in Maryland, and as well in Baltimore.

Although, it is technically possible for the implementation of road pricing, the decision on whether road pricing should be implemented relies on whether it is cost effective. The cost of the any road pricing scheme, which includes the implementation, operation and maintenance cost, is relatively easy to estimate and could be based on the experience of other similar scheme or from the available budget. But the benefits (for example the expected saving in travel time) is relatively complicated to be estimated as it highly depends on the traveler's behavior and network equilibrium. Thus, before any actual implementation, various network equilibrium models have to be considered for analytically evaluate and compare the benefits of various road pricing schemes in order to find the most socially beneficial scheme. In this study, the continuum modeling approach is proposed for the feasibility study of road pricing in Baltimore Metropolitan Region. The continuum modeling approach is mainly used for the initial phase of planning and modeling in broad-scale regional studies, in which the focus is on the general trend and pattern of the distribution and travel choice of users at the macroscopic, rather than at the detailed, level. In the continuum approach, the dense network is approximated as a continuum in which users are free to choose their routes in a two-dimensional space. The fundamental assumption is that the differences in modeling characteristics, such as the travel cost and the demand pattern, between adjacent areas within a network are relatively small as compared to the variation over the entire network. Hence, the characteristics of a network, such as the flow intensity, demand, and travel cost, can be represented by smooth mathematical functions (Vaughan, 1987).

In the feasibility study of road pricing, as it mainly focuses on the macroscopic effect of road pricing schemes on travelers' choice and behavior with limit input information available at this stage, the continuum modeling approach has a number of advantages over the traditional network modeling approach (Blumenfeld, 1977; Taguchi and Iri, 1982; Sasaki et al., 1990; Gwinner, 1998). The most significant advantage of continuum model in road pricing study is its ability to visualize, in a two dimensional sense, the influence of the adopted road pricing scheme on the spatial variation of the model variables, like the distribution of demand and flow. With the ability to visualize the spatial variables, the continuum approach could also be used in designing and choosing the shape of charging cordon for the implementation of road pricing (Ho *et al*., 2005). Besides the ability to visualize spatial interaction and design of cordon shape, the continuum model also have the following strengths in macroscopic and large-scale regional studies. First, it reduces the problem size for the dense and large-scale transportation networks, because problem size in the continuum model depends on the method that is adopted to approximate the modeling region but not on the actual network itself, and thus an effective approximation method, such as the finite element method, can extensively reduce the size of the problem. This reduction in the problem size also means savings in computational time and memory. Second, less information is required in setting up in a continuum model. As a continuum model can be characterized by a small number of spatial variables, it can be set up with a much smaller amount of information as compared to the traditional network modeling approach, which needs information for all of the included links. This makes the continuum model suitable for macroscopic studies in the initial phase of design, because the collection of data in this phase is time consuming and labor intensive.

When considering the real-world application, the more precise and complete the information/data adopted, the more accurate will the result of the model be. But acquiring information/data is a time consuming and expensive process, thus there is always a tradeoff between the accuracy of the model and the cost for acquiring its input data. By performing the sensitivity analysis, one can identify the information/data that the model are sensitive to and could allocate more resources in acquiring them for ensuring the accuracy of the model. This paper is organized as follows. After introducing the basic continuum formulation of road pricing problem and the evaluation procedure for road pricing study in Baltimore Metropolitan Region in the next section, the case study of the road pricing in Baltimore Metropolitan Region will be introduced and solved in Section 3. Section 4 completes the sensitivity analyses for the proposed model and some interpretations and discussions of the results will be included. Finally, conclusions are presented in Section 5.

2. Continuum Model and Evaluation Procedure

In this section, the concept of the continuum modeling approach used in road pricing will be introduced in section 2.1 and the evaluation procedure for the road pricing study in Baltimore Metropolitan Region, which based on the continuum model, will be describe in section 2.2.

2.1 Continuum Formulation of Road Pricing Problem

In this study, the continuum approach for modeling the road pricing problem introduced in Ho *et al*. (2005) was adopted. In Ho *et al*. (2005), the road pricing problem is solved and evaluated by considering three different models: i) User-equilibrium model, which replicates the current non-toll condition; ii) System-optimal model, which determines a location-dependent toll charge pattern that maximize the social benefit of the system; iii) Cordon-base model, which based on the results of the system-optimal model to solve for more practical cordon charging schemes. In this paper, these three models will be adopted with model parameters redefined and calibrated for suiting the current conditions in Baltimore Metropolitan Region. The elastic demand and transportation cost function adopted will be discussed in more details in the following text.

The demand function, which defines the demand generated per hour per sq. mile at location (x,y), takes the following form:

$$q(x,y,u) = D\exp(-ku) \tag{1}$$

where D is the potential demand which represents the maximum number of citizens would choose to travel to work during the peak hour. In this study, this quantity is estimated by the work force at that unit area. u is the total travel cost at location (x,y). This cost includes two components: the transportation cost, which accounts for the time taken for the trip, and the toll (if any) they have to pay in this trip. k is the elasticity parameter and will be calibrated with the demand at CBD in the user-equilibrium model. For the transportation cost function, it defines the cost per unit distance of travel at location (x,y) and takes the following BPR-liked form:

$$c(x,y,\mathbf{f}) = v_t\left(t_0 + s_t|\mathbf{f}|^{\beta}\right) = a + b|\mathbf{f}|^{\beta} \tag{2}$$

where $\mathbf{f}$ is the flow vector in travelers per mile at (x,y). The direction of this vector represents the direction that the travelers taken for them to travel from their home location the CBD in this two-dimensional continuum model; v_t is the value of time for the travelers; t_0 is the free flow travel time which is evaluated from the free flow speed; and s_t is the congestion sensitivity parameter which depends on the road network density, average number of lane, average lane capacity, free flow travel time and the parameter α of the following typical BPR function:

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$$t = t_0\left[1+\alpha\left(\frac{V}{C}\right)^{\beta}\right] \tag{3}$$

The parameter β of the BPR function will also adopted in Equation (2) to account for the non-linearity of the cost-flow relationship. In this study, the transportation cost function only includes the travel time but it could be easily extended to include other quantities like fuel cost. By considering the above discussed elastic demand and transportation cost function, similar optimization models for the user-equilibrium, system-optimal and cordon-based charging models introduced in Ho *et al.* (2005) could be formulated. For the adopted system-optimal model, the following social benefit is maximized:

$$B_{\mathrm{E}}(\mathbf{f}) = \iint_{\Omega}\int_0^{q} D^{-1}(\xi)\,\mathrm{d}\xi - c|\mathbf{f}|\,\mathrm{d}\Omega - \sum_n C_n Q_n \tag{4}$$

where D^{-1} is the inverse demand function; Q_n is the demand at CBD n; C_n is the extra cost at CBD n (e.g. walking time, delay etc) that the travelers experience after they arrived their destination region. The first term of this social benefit function defines the total traveler benefit (e.g. wages, satisfaction of needs, etc) that the travelers gain as they make the trip while the last two terms govern the costs of the system. Although travelers have to pay toll as they travel, it is not included in equation (4) as paid toll is transferable and the toll collected could be used beneficially to the whole system. At the optimal point of this system-optimal model, it could be proven that all travelers have to pay for a location-dependent toll rate τ (\$/mile), such that:

$$\tau(x,y) = \gamma b|\mathbf{f}|^{\gamma} \tag{5}$$

By charging this toll rate to all travelers, the user-equilibrium travel pattern under this toll will be equivalent to system-optimal travel pattern that maximized the social benefit. By integrating this toll rate along the path taken by the travelers, the paid toll, T (\$), that a traveler has to pay for his trip is defined as follows:

$$T(x, y) = \int_p \tau \, \mathrm{ds} \tag{6}$$

where p is the path taken by the traveler. Noted that as this paid toll value is dependent on the path taken, its values will vary over the modeling region and forms a paid toll pattern. The detail proofs for the user-equilibrium and system-optimal condition for this continuum model will not be included in this paper, but a similar and more detail proof could be found in Ho *et al.* (2005). By using the finite element method (Zienkiewicz and Taylor, 1989) for approximating the continuum nature, the solution of the continuum models could then be searched by using Newtonian algorithm with step size determination.

Although the toll rate defined in equation (5) could maximize the social benefit, its location dependent nature makes it difficult to implement (as it requires a GPS-based charging system) and to be understood by the public (as the toll rate changes continuously over the modeling region). Owning to these reasons, the less optimal, but more practical and understandable cordon-based charging scheme, which travelers will only be charged as they passing across the cordon, is adopted. In the continuum model, the shape of these charging cordons will be taken as the iso-contours of the paid toll (T) pattern from the system-optimal case (Ho *et al*., 2005). The number and location of the contours that are used to setup the cordon-based charging scheme will be chosen by system planner such that it is the most beneficial and feasible to the system based on the demand, toll collected and the social benefit estimate by the cordon-based models. As only finite number of charging cordons (iso-contours) are consider in any of the cordon-based charging scheme, the paid toll level of the iso-contours found from the system-optimal case should be sub-optimal and need to be further optimized for maximizing the social benefit (Ho *et al*., 2005)

2.2 Evaluation Procedure for the Road Pricing Study for the Baltimore Metropolitan Region

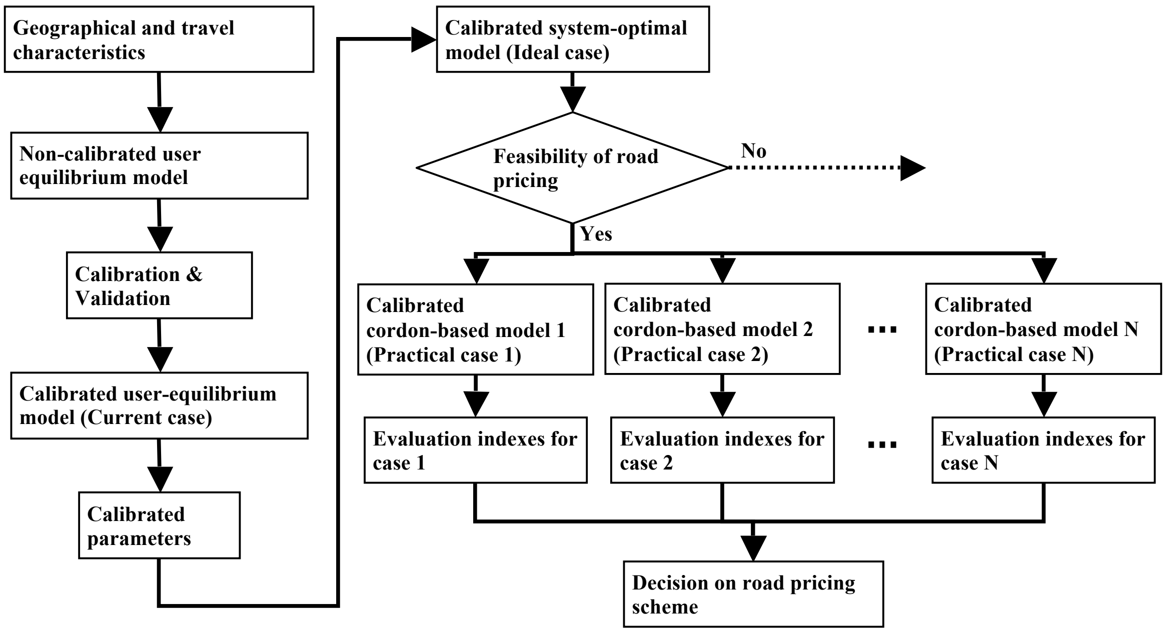

Figure 1. Flow chart of the proposed road pricing study in Baltimore Metropolitan Region

The flow chart in Figure 1 gives the proposed general procedure, which based on the continuum models discussed in the previous section and Ho *et al.* (2005), for the road pricing study in Baltimore Metropolitan Region. The whole study could be summarized in the following steps:

1) Geographical and travel characteristics (such as demand, travel cost function, map of the region, etc.) of the Baltimore Metropolitan Region is collected and used to setup the non-calibrated user-equilibrium model.
2) The non-calibrated user-equilibrium model is solved, calibrated and validated with the collected data (such as hourly demand at CBDs, screen line traffic count, etc.). The calibrated model is considered as the base model that reflects current situation.
3) With the calibrated parameters, the system-optimal model is formulated and solved. This system-optimal model is considered to be an ideal case as it maximized the social benefit.
4) The feasibility of road pricing is then evaluated by considering the gain in social benefit in the ideal case, when compare with the user-equilibrium case (i.e. the current case), and the expected cost/budget for implementing the road pricing scheme. If the gain in social benefit is larger than the cost, then road pricing in Baltimore Metropolitan Region is considered to be potentially feasible and is worthwhile to generate more cordon-based

charging schemes in the next step for more detail analysis. Otherwise, the road pricing is considered to be not feasible. Social benefit of the ideal case is used in the assessment of road pricing feasibility as it gives the maximum achievable benefit by implementing road pricing in this region (i.e. the first-best solution in Ho *et al.*, 2005). Thus if the result of this ideal case is not cost effective, it is not possible for the cordon-base charging, which is the second-best solution (Ho *et al.*, 2005), derive from this solution would be cost effective

5) If the system-optimal case is found to be cost effective, the procedure is then proceeded to generate different cordon-based charging scheme for further evaluation. By taking the iso-contours of the paid toll pattern from the ideal case as charging cordons, different cordon-based charging schemes are setup and solved.
6) Evaluation indexes (such as total demand, social benefit and toll collected) of those cordon-based charging schemes are found and presented to the system planned for comparison and decision making

3. The Case Study

In order demonstrate the potential applicability of the continuum modeling approach and as a base scenario for the sensitivity analysis in the next section, the continuum models for the Baltimore Metropolitan Region is setup and solved. Figure 2 shows the Baltimore Metropolitan Region that will be used as the modeling region throughout this study. In this region only one CBD, which is located within the Baltimore City, is considered and is represented by the black circle. In general the shape of the CBD could be taken in any forms based on the distribution of trip ends of that region, but in the current study, due to the lack of more detail information, a circular shape is assumed. The parameters that are used in this model could be found in Table 1 below and the quantities that are adopted in this base scenario are shown in the third column of the same table.

Figure 2. The Baltimore Metropolitan Region

Table 1. List of input parameters considered in the sensitivity test

Input parameters	Description	Value taken in the base scenario
S1	Labor force within Baltimore Metropolitan Region	1,291,461[a]
S2	Area of the Baltimore Metropolitan Region (sq. mile)	2,172[b]
S3	Peak hour duration (hour)	2[c]
S4	Road network density in Baltimore Metropolitan Region (mile/sq. mile)	2.737[d]
S5	Average number of lane	3[c]
S6	Average lane capacity (veh/hour)	1000[c]
S7	Free flow speed (mile/hour)	50[c]
S8	Value of time (USD/hour)	14[e]
S9	Parameter in the BPR function (α)	0.17[e]
S10	Parameter in the BPR function (β)	4[e]

S11	Trip ends at the Baltimore City	89,753[a]
S12	Extra cost at CBD (USD)	10[c]

Sources: [a] Baltimore Metropolitan Council website, 2007 <http://www.baltometro.org/index.htm>

[b] Base on the measurement of the developed continuum modeling region in Figure 2

[c] Model assumptions

[d] Federal Highway Administration, 1992. Report on the 1992 U.S. Tour of Europe Concrete Highways.

[e] Baltimore Region Travel Demand Model Version 3.3 2000 Validation: Task Report 07-8.

Based on the geographical characteristics shown in Figure 2 and the input parameters in Table 1, the road-pricing study in Baltimore Metropolitan Region is setup and solved. The elasticity parameter k of the user-equilibrium model is calibrated base on the hourly trip ends at Baltimore City found by dividing S11 with S3. Some of the numerical results for the user-equilibrium, system-optimal and cordon-based charging solutions for this case study are shown in Table 2 below.

Table 2. Comparison of the user-equilibrium and system-optimal results of the base scenario

	User-equilibrium case	**System-optimal case**	**Cordon-based charging case**
Total demand	44,859	34,664	37,149
Total toll collected ($)	-----	85,463	76,195
Total social cost ($)	786,246	567,982	574,252
Social benefit ($)	248,019	269,013	265,831

Comparing the social benefits from the user-equilibrium case, which represents the current situation, and the system-optimal case, which represents the scenario that the travelers of Baltimore Metropolitan Region is charge with a spatially variated toll rate, the gain in social benefit due to the implementation of congestion pricing scheme is $20,994 per peak hour of work day. By considering a 2-hour peak hour period and 5 work days a week, the annual gain in benefit is approximately $11 million. Thus, if the expected implementation, operation and maintenance cost of road pricing implementation in Baltimore Metropolitan Region is less than this value, then road pricing is considered as potentially feasible and is worthwhile to generate different more practical cordon-based charging scheme for more detail evaluation

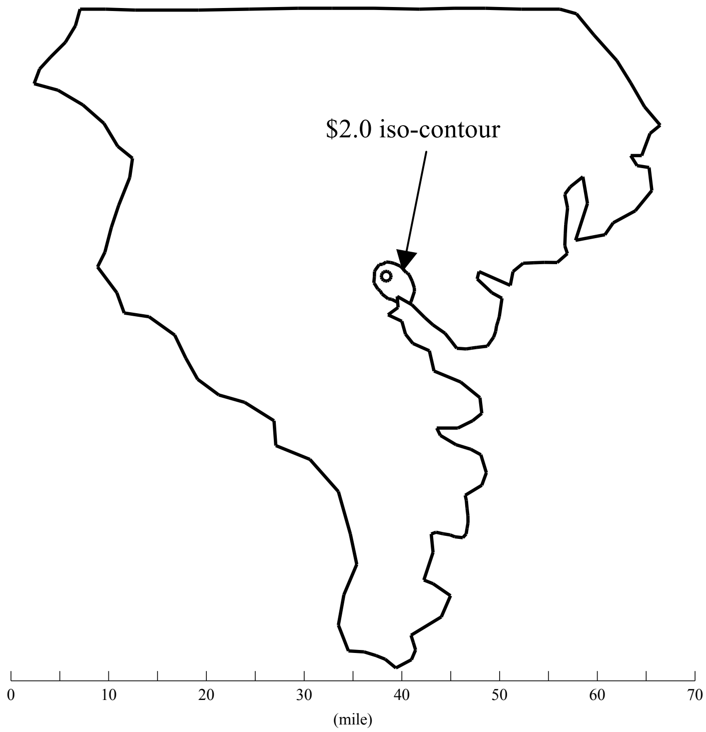

Figure 3 Location and shape of the $2.0 iso-contour

Comparing the results in Table 2, it could be seen that the demand in the system-optimal case reduces. This reduction in demand could be explained by the additional cost of travel (i.e. the toll imposed) on the elastic demand function defined in equation (1). In reality, this reduction of demand could be interpreted as the shifting of travelers that are originally choose to travel by automobile during the peak hour to make their trips in other time or by other mode due to the introduction of toll. Although the reduction in demand will decrease the total traveler benefit (as less travelers are served in this period), this decrease could be compensated by the reduction of the total cost (as there is smaller delay due to less congested network) and resulted in an increase in the social benefit of the system.

By considering the paid toll (T) pattern of the system-optimal solution, iso-contours of different toll level (e.g. $1.0, $1.25) could be chosen by the system planner to setup different cordon-based charging model. In this study, the $2.0 iso-contour (as shown in Figure 3) is used as an example and the corresponding cordon-based model is setup and solved. The toll level of this cordon is re-optimized for maximizing the social benefit under this cordon-based charging scheme. It is found that by charging \$2.11 at this cordon, the social benefit is maximized and gives the result in the last column of Table 2. Although the total toll collected and the social benefit is less than that in the system-optimal case, this scheme provides a more practical way of implementing of road pricing in Baltimore Metropolitan Region. Under this charging scheme, the peak hour demand has been reduced by 7,710 as comparing to the user-equilibrium case. This difference should be considered by the system planner in comparing different cordon-based schemes to ensure this shifted demand could be served by the other mode of transport within the region. A similar but more detail explanation for the results for the user-equilibrium, system-optimal and cordon-based charging cases could be found in Ho *et al.* (2005).

4. Sensitivity Analysis

In order to find out how the accuracies of the input parameters affect the results of the proposed road pricing model for the Baltimore Metropolitan Region, sensitivity analyses of those input parameters on model outputs are performed. In this analysis, the sensitivities of model outputs with respect to the 12 input parameters listed in Table 1 will be studied. For each of the input parameter, four different cases of ±20% and ±5% change to the value adopted in Table 1 are considered for representing different levels of inaccuracy. For each case, the demand elasticity parameters will be re-calibrated with respect to the trip ends at the Baltimore City. With the re-calibrated parameter, the user-equilibrium and system-optimal solution of that case will be solved and the results of the system optimal case will be compared with that of the base scenario, which are the results of the case study presented in section 3.

In this sensitivity analysis, system-optimal case, instead of the cordon-based charging case, is chosen for comparison. As discussed in the end of section 2.1, although the shapes of the charging cordons in the cordon-based charging case are defined based on the iso-contours of the paid toll (T) pattern solved from the system-optimal case, the number and locations of cordons is chosen by the system planner for the most beneficial and feasible to the system based on the demand, total toll collected, social benefit and the estimated implementation cost of that cordon scheme. Also, for the same paid toll pattern solved from the system-optimal case, there are infinite combinations of iso-contours that could be taken as the potential cordon-based charging scheme. Thus it is not possible to compare each (or some) of these cases to the base scenario before any choice of cordon scheme. Therefore the system-optimal cases, which the cordon-based case is based on, for different parameter inaccuracies are chosen to compare with the system-optimal case in the base scenario. In this sensitivity analysis, the effect of the inaccuracies of input parameters will be compared in terms of the evaluation indexes and the shape of charging cordon.

4.1 Sensitivity of evaluation indexes

In this section, the sensitivity of total demand, total toll collected and social benefit of the proposed system-optimal road pricing model with respect to the 12 input parameters will be considered and discussed. These indexes are chosen due to their importance in evaluating the road pricing schemes. More detail discussion on the importance of these indexes and the change for each of these indexes with respect to the inaccurate parameters input could be found in the following subsections.

4.1.1 Total demand

Total demand of the system is one of the important indexes of the road pricing study. It is because, by knowing the total demand before and after the implementation of road pricing, the number of citizen that is driven out of the system is known. Based on this figure, the system planner has to provide other mode of transport, such as bus, to maintain the mobility of the whole system.

Figure 4 and 5 are respectively the plots for the change of demand with respect to a 20% and 5% inaccuracy in the estimation of input parameters. Considering these plots, it could be easily seen that the proposed procedure for road pricing study does not amplified the inaccuracies in estimating the total demand (i.e. an 20% inaccuracy in the input parameter will only results in a similar, or less, percentage change in the estimation of total demand). Also, the calibration process could effectively compensate most of the inaccuracies in the input parameters, as in most of the case the percentage change in the total demand is far less than the inaccuracy in the input parameters. The major reason of having this effective calibration process may due to the close relation between the demand elasticity parameter and the demand for the user-equilibrium and system-optimal solution.

From Figures 4 and 5 it could be seen that the total demand is significantly affected by the inaccuracies in peak hour duration (S3) and trip ends at the Baltimore City (S11) when compare with the other input parameters. This is because by dividing S11 with S3 the hourly trip ends at Baltimore City could be found and is used in the model calibration. Thus changing those parameters will extensively affect the effectiveness of calibration and cause a relatively large discrepancy with the base scenario.

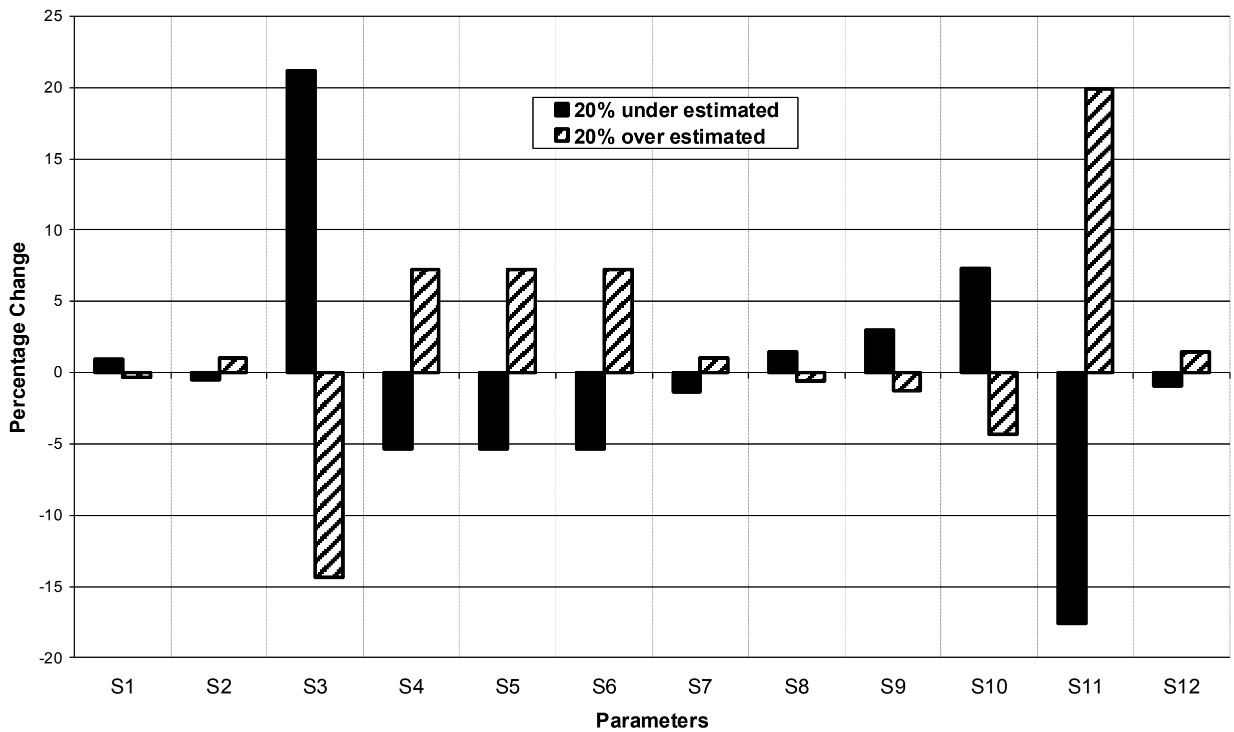

Figure 4. Percentage change of demand for the system optimal solutions with 20% inaccuracy in input parameters

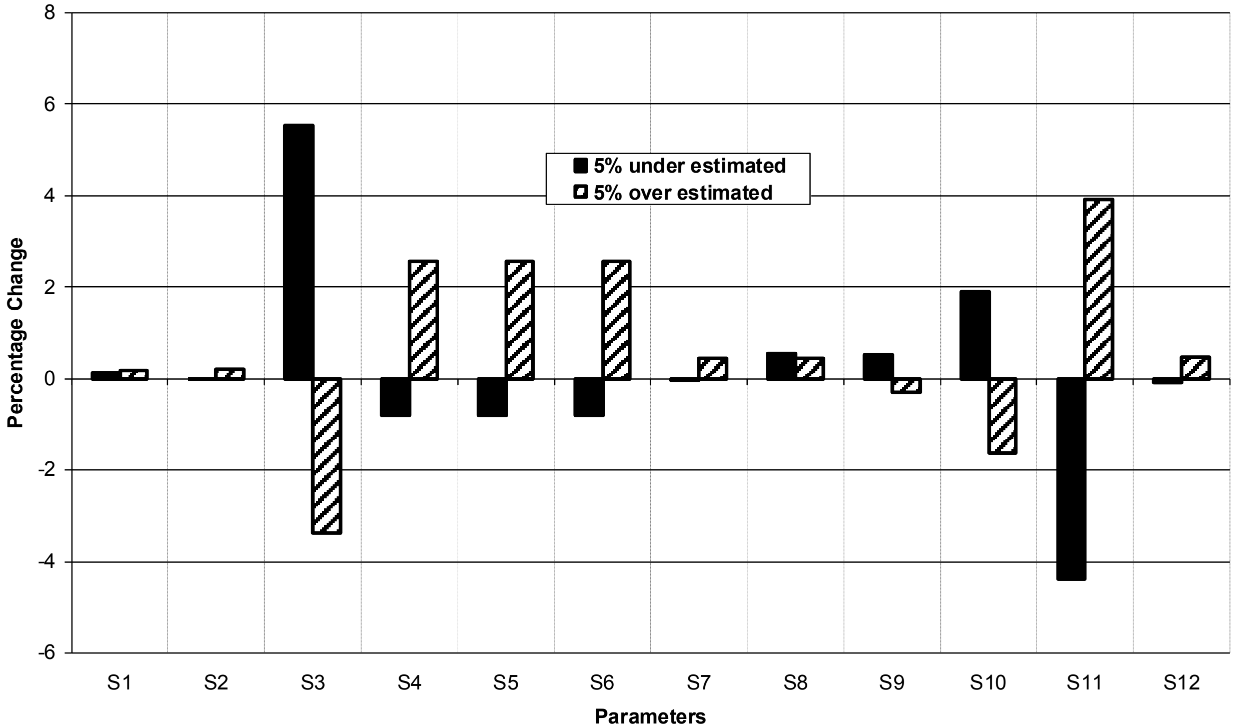

Figure 5. Percentage change of demand for the system optimal solutions with 5% inaccuracy in input parameters

4.1.2 Total toll collected

In the implementation of the any road pricing scheme, one of the most important thing is to ensure the toll collected by this scheme could cover the operating and maintaining cost of the road pricing system and as well to pay back some of the implementation cost. Thus, the total toll collected by any proposed road pricing scheme should be considered as one of the important indexes for evaluating the feasibility of that road pricing scheme. Figure 6 and 7 are respectively the plots for the change of total toll collected with respect to a 20% and 5% inaccuracy in the estimation of input parameters. In Figure 6, it could be seen that the total toll collected is sensitive to parameters S3, S4, S5, S6, S10 and S11. For a 20% inaccuracy in those input parameters, the change of toll collected, when comparing with the base scenario, is varying from a 58% decrease to a 173% increase.

The reason of having such extensive change in the total toll collected is mainly due to the definition of toll in the proposed system-optimal model. In the system-optimal model, the toll charged at each point is defined by Equation (5), which is dependent on the volume ($|\mathbf{f}|$) and capacity-related parameter (b) of that location. As this suggested toll level is non-linearly related to the volume to capacity ratio by the parameter β (which is normally larger than 1) the inaccuracy in estimating the volume- and capacity-related parameters will be amplified. As S3 and S11 are used to calibrate the demand in the user-equilibrium case, underestimating S3 and overestimating S11 will overestimate the demand and volume for the system-optimal case. On the other hand, as the road network density (S4), average number of lane (S5) and average lane capacity (S6) are directly used in the estimation of the capacity for a particular point within the system, underestimating these parameters will underestimate the capacity of that particular area. By the definition of toll in equation (5), overestimating the demand and underestimating the capacity will enormously increase the toll needed for maximizing the social benefit in the system-optimal case. This increase in toll level overrides that of the decrease of demand due to the higher travel cost and result in an overall increase in total toll collected. Similar interpretation could be applied for the cases of 5% inaccuracy in the input parameters (Figure 7).

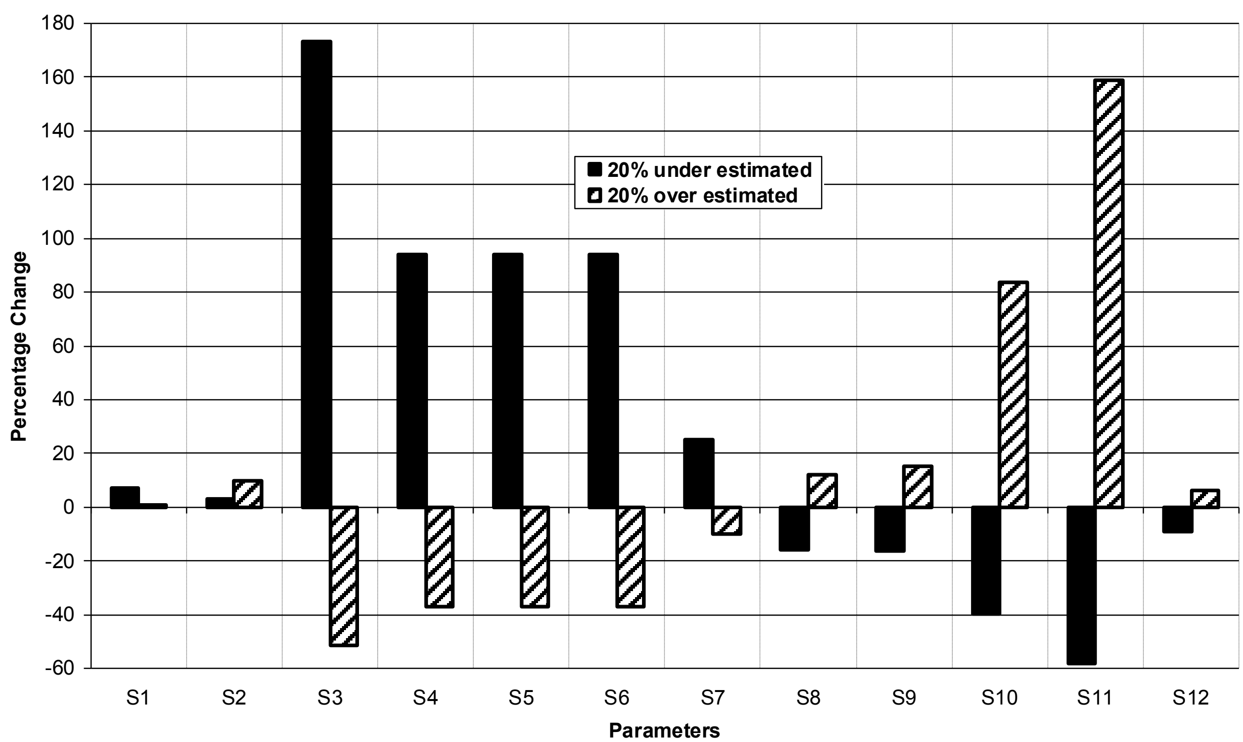

Figure 6. Percentage change of total toll collected for the system optimal solutions with 20% inaccuracy in input parameters

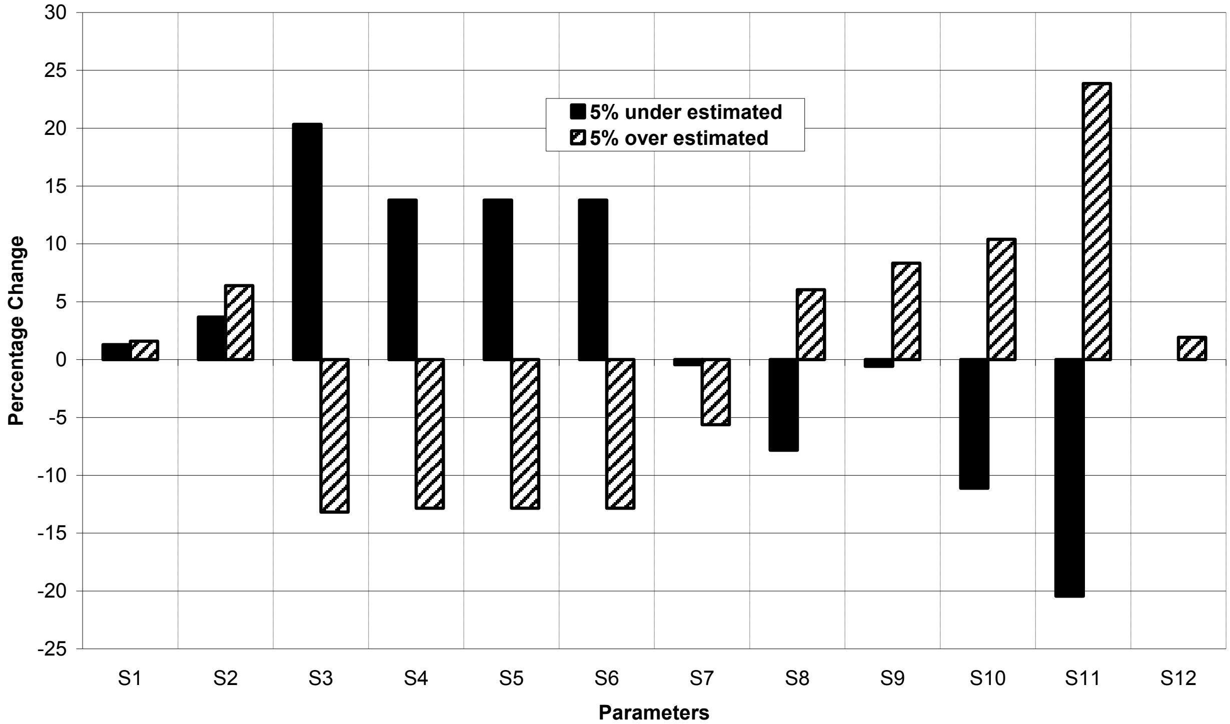

Figure 7. Percentage change of total toll collected for the system optimal solutions with 5% inaccuracy in input parameters

4.1.3 Social benefit

Besides the total demand and total toll, social benefits before and after the implementation of road pricing scheme is also an important indicator for the system planner to evaluate the overall effectiveness of that scheme and to compare between different schemes. Figure 8 and 9 are respectively the plots for the change of social benefit with respect to a 20% and 5% inaccuracy in the estimation of input parameters. Comparing these two plots with the ones for the total toll collected, it could be seen that they are quite similar as both of them are sensitive to the parameters S3, S4, S5, S6, S10 and S11. But, the actual effect of the inaccurate input parameters on the change of social benefit is difficult to summarize into some simple mechanisms or relations as the social benefit is a composite function depends on the total traveler benefit, which is the inverse demand function, and the total system cost, which depends on the unit transportation cost function and the total demand. Varying any of these parameters will cause different effects on those functions and quantities, and results in a combined effect in the social benefit.

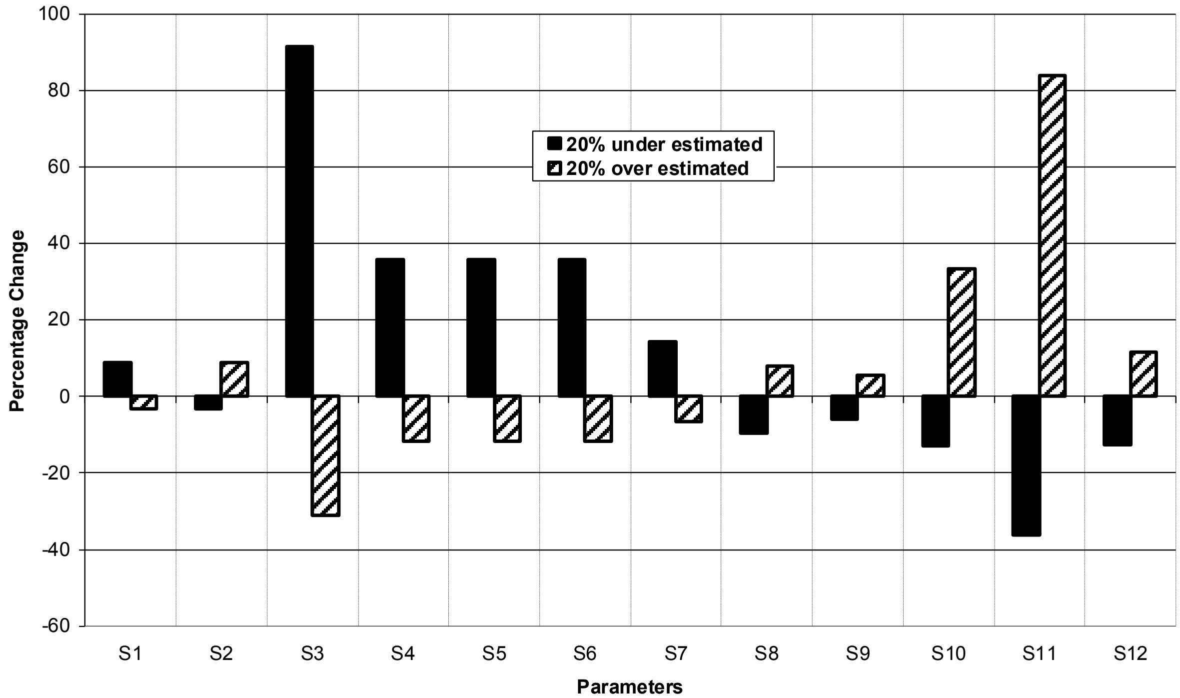

Figure 8. Percentage change of social benefit for the system optimal solutions with 20% inaccuracy in input parameters

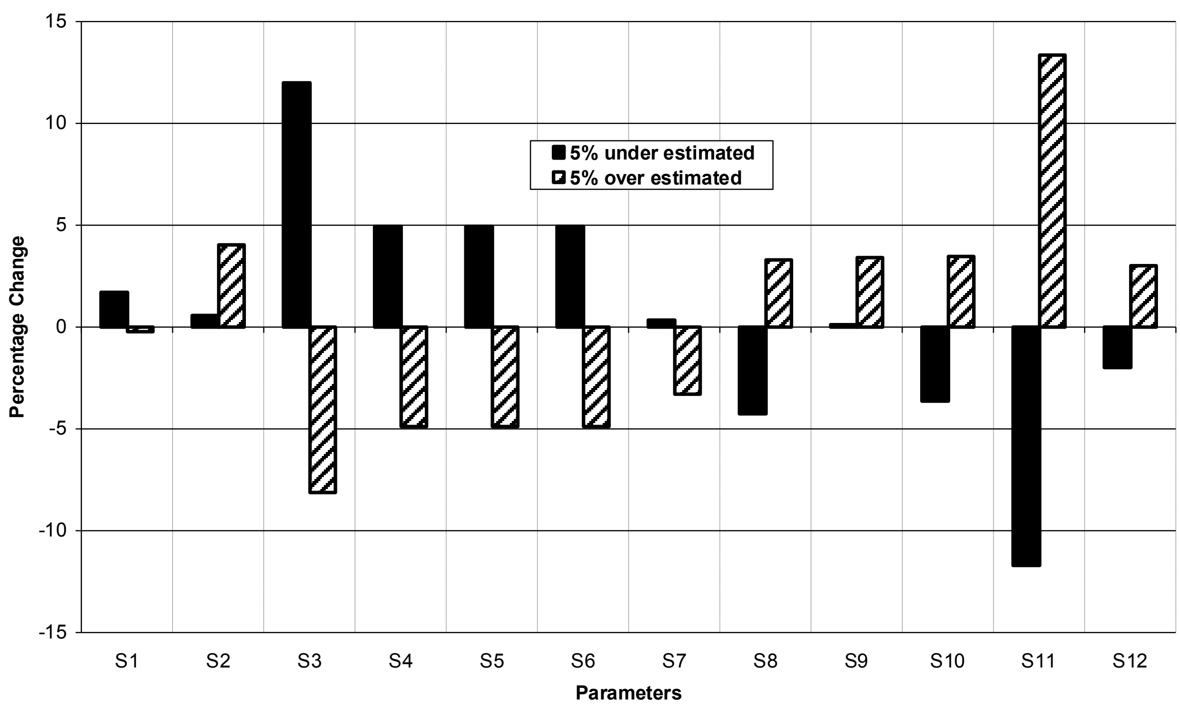

Figure 9. Percentage change of social benefit for the system optimal solutions with 5% inaccuracy in input parameters

Some remarks for the sensitivity of evaluations indexes:

a) The peak hour duration (S3) and the trip ends at Baltimore City (S11) are the most sensitive parameters for the three evaluation indexes considered, as they are used in the model calibration.
b) Road network density (S4), average number of lane (S5), average lane capacity (S6) and the non-linear parameter (β) of the BPR function (S10) are also the sensitive parameters for total toll collect and the social benefit as these parameters will directly affect toll charged in the system-optimal case.
c) Comparing with the parameters in a) and b), the evaluation indexes are less sensitive (or at least not excessively amplifying the inaccuracy) to the remaining parameters.

4.2 Sensitivity of paid toll pattern

In the previous section, the evaluation indexes: total demand, total toll collected and social benefit are considered. Although these indexes are important in the evaluating the effectiveness of a road pricing scheme, they are aggregated quantities and could not precisely shows the impact of the accuracy of input data on the cordon shapes suggested by the system-optimal model and the choice of cordon by system planners.

In this proposed procedure, cordons in different cordon-based charging schemes are setup by choosing different iso-contours from the paid toll (T) pattern solved from the system-optimal case. Therefore, the sensitivity of the cordon design, in terms of shape and location, with respect to the inaccurate input parameters could be considered as the sensitivity of the paid toll pattern for the system-optimal case with respect to the inaccurate input. In this sensitivity test, focus will be on whether the paid toll pattern will be substantially changed due to the inaccurate input parameters. For example, one should check whether an originally east-west oriented elliptic iso-contour will changed to north-south oriented due to the inaccurate input parameter.

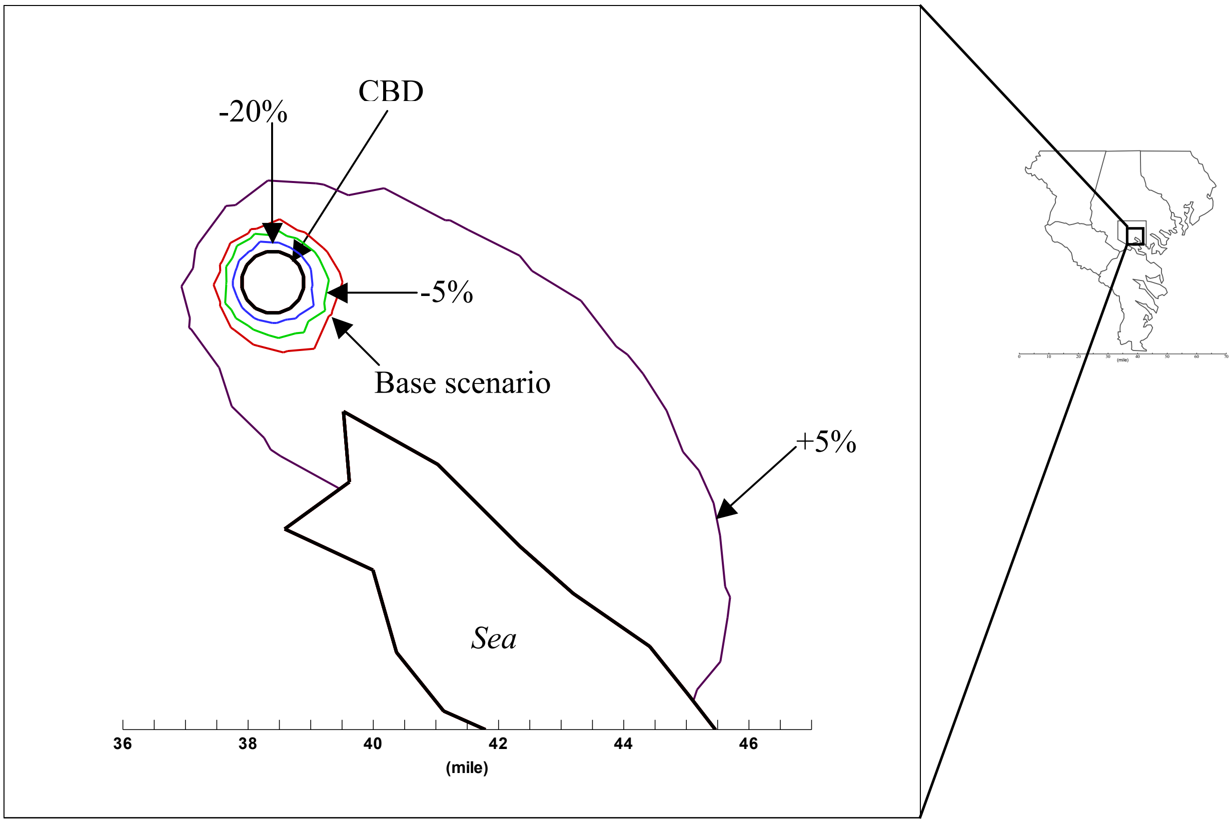

Figure 10. Plot of the $1.75 iso-contour line of the total paid toll from the system-optimal case for the different level of inaccuracy in S4, S5 and S6

One of the attempts to find the sensitivity of the paid toll pattern on inaccurate input parameter is to compare the iso-contours with same paid toll level from the system-optimal case. In this case, the $1.75 iso-contour is chosen as an example, but different iso-contours could also be used and share the similar interpretation below. Figure 10 shows the $1.75 iso-contours of the paid toll from the solution of the system-optimal case for different level of inaccuracy in S4, S5 and S6. The innermost circle in Figure 9 is the boundary of CBD with zero paid toll and the paid toll increases as moving away from this boundary. This figure is identical for S4, S5 and S6, as these parameters are used similarly in estimating the capacity. It could be seen that the shape and location is quite different for the +5% error case and the $1.75 iso-contour even does not exist for the +20% error case. This huge difference of shapes and locations is caused by the substantially underestimated paid toll, which is resulted from the overestimation of capacity due the increase of S4, S5 and S6. Although it is quite different in the shape of these iso-contours with same toll level, it is not sufficient to conclude that the paid toll pattern is sensitive to the inaccurate parameters input as these difference may only caused by the different in the estimated paid toll level, but not the pattern itself. Instead, the existence of iso-contours with similar shape (although may be different in paid toll level) for different parameter inaccuracies should be found for evaluating the sensitivity of paid toll pattern.

Figure 11 shows a plot of the similar iso-contours of the paid toll (*T*) pattern around the CBD (the innermost circle) for different levels of inaccuracy in input parameter S4, S5 and S6. As in the continuum modeling approach, the model characteristics is assumed to be continuously variated and could be represented by some smooth mathematical function (Wong *et al.* 1998 and Ho *et al.* 2006), the existence of these similar iso-contour lines suggested that these cases of different parameter inaccuracies share a similar paid toll pattern. Noted that, although these cases have a similar paid toll pattern, their toll levels are different. The toll levels for each of the contours in Figure 10 are: Base scenario – \$2.00; -20% error case – \$4.13; -5% error case – \$2.33; +20% error case – \$1.2; and +5% error case – \$1.72. The existence of these similar iso-contours suggested that it is possible for the system planner to pick cordons with similar shape, although with different toll level, as the candidate for cordon-based charging scheme regardless of the inaccuracy of the parameters. The difference in toll level is immaterial as it will be re-optimized to find the most system beneficial toll level in the cordon-based charging scheme under the current accuracy of input data. Similar plots and explanation as for Figure 10 and 11 could also be applied for the other input parameters. To sum up, although the estimated toll level will affected by the inaccurate input parameters, the paid toll pattern (i.e. the shape for the potential cordon) is not sensitive to those inaccuracies.

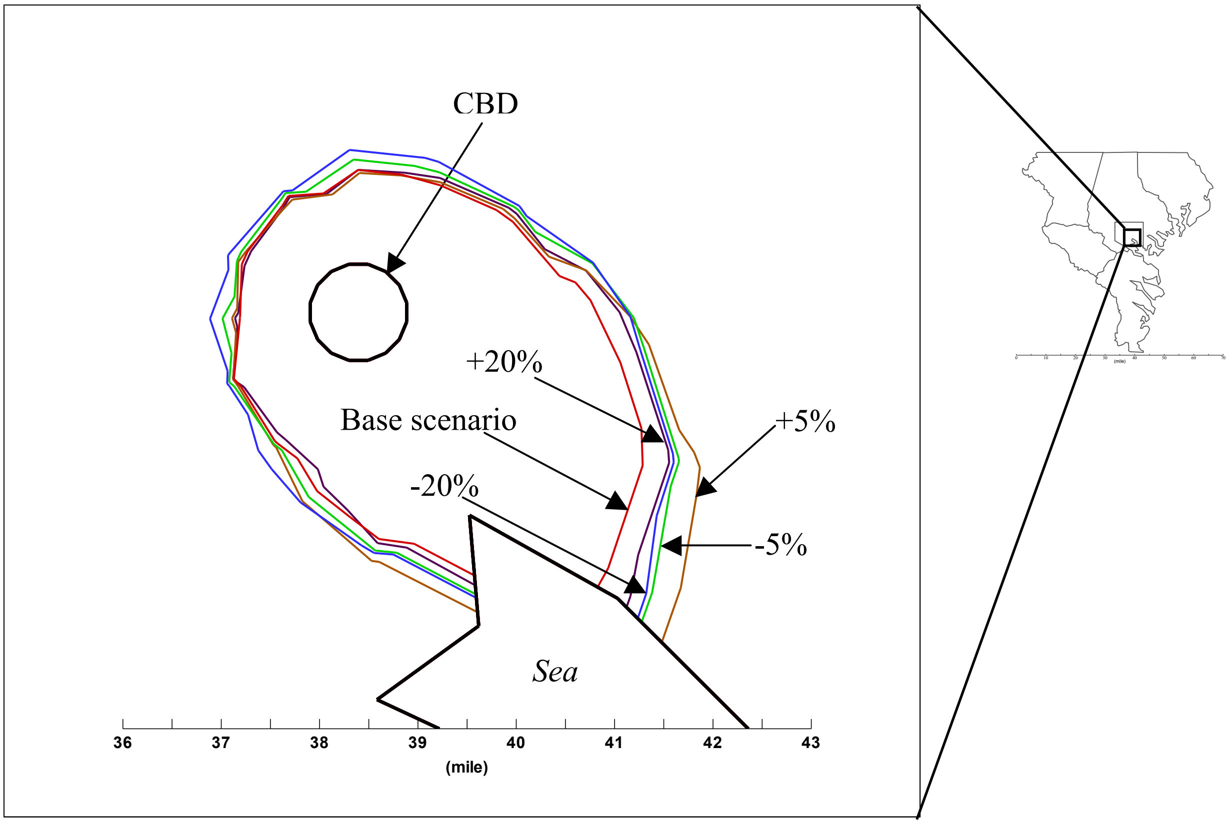

Figure 11. Plot of the similar iso-contour lines of the total paid toll from the system-optimal case for the different levels of inaccuracy in S4, S5 and S6

5. Concluding Remarks

In this paper, the potential of real-world application of continuum models in road pricing study is demonstrated by using a case study in Baltimore Metropolitan Region. In this case study, an evaluation procedure is proposed by using the continuum models as the basic evaluation blocks. This evaluation procedure could be summarized as follows:

1) Build, calibrate and validate the user-equilibrium model (current case)
2) Solve the system-optimal model (ideal case)
3) Determine feasibility based on the social benefit gain (by comparing the current and the ideal case) and the expected cost of implementation
4) Different cordon-based schemes are setup and solved based on the result of the

system-optimal model

5) Social benefits, demands, total toll collected and expected implementation cost for different cordon-based schemes are presented to system planner for choosing the most beneficial and feasible scheme.

Apart from introducing the procedure for conducting road pricing study by the continuum model, this study also completed the sensitivity analysis for addressing the quality requirement of the input parameters. The sensitivity analysis of total demand, total toll collected, social benefit and paid toll pattern of the road-pricing problem for the Baltimore Metropolitan Region is completed for 12 demand-, geographical- or cost-related input parameters used for model building. It is found that the proposed model is most sensitive to peak hour duration and trip end at Baltimore City as these are the parameters used in model calibration. Besides, this model is also sensitive to road network density, average number of lane, average lane capacity and the non-linearity parameter of the cost function as the inaccuracies of these parameters are amplified by the non-linearity of the travel cost function. For the total paid toll pattern, it is found that, although the toll level is different, similar pattern exists for different inaccuracy level of the tested parameters.

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