Optimal Detector Locations for OD Matrix Estimation

Ying Liu[1], Xiaorong Lai[2], Gang-len Chang[3]

Abstract

This paper has investigated critical issues associated with Optimal Detector Locations for OD matrix estimation, including a discussion of limitations embedded in existing models and heuristic algorithms. Grounded on the core methodology of existing literature (Yang, 1998), this paper has proposed a heuristic algorithm for identifying the optimal set of detector locations under a given budget constraint for effective OD matrix estimation. The algorithm tries to simultaneously optimize OD coverage, net OD flow intercepted and link-OD flow fraction. Our numerical experiment results have indicated that the proposed algorithm is quite promising for potential applications.

Background and Problem Nature

As is well recognized, OD matrix, depending on its nature, plays a key role in both urban planning and traffic control. Since the actual distribution of traffic demands is extremely difficult to obtain in practice, the methodology of estimating OD matrix from traffic counts has received increasing attention over past decades. Examples of methods for such applications include Least Square models, Entropy Maximization models, Likelihood Maximization models, etc. (Bell, 1991)

Due to the budget constraints in practice for collecting traffic volumes at every link and intersection approach, the estimation quality of existing methods depends significantly upon the locations of traffic counting stations in the network (Lam, 1990; Yang, 1991). Thus, how to identify the optimal set of locations for OD estimation under the budget constraints has emerged as one of the imperative research and implementation issues.

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This critical problem can be stated specifically as follows. Based on certain assignment rules or models, the true OD matrix $\boldsymbol{M_T}$, with its entry m_{ij}^{T} indicating the actual traffic demand from origin i to destination j, is loaded on the network. Accordingly, each link a obtains a traffic count v_a. Using a subset of these traffic counts $\{v_a, a \in L\}$ as constraints, an estimated OD matrix $\boldsymbol{M_E}$ can be solved with existing OD estimation methods. The core research issue in this paper is how to identify such a subset of links L, $|L| \le l_o$, so that the quality of $\boldsymbol{M_E}$ can be optimized.

Note that the complexity of this critical issue is compounded by the fact that we generally do not have a true OD matrix $\boldsymbol{M_T}$ for any real-world network for comparison. Thus how to define the quality of an estimated OD matrix $\boldsymbol{M_E}$ is also a challenging task. A research to directly integrate OD estimation uncertainty is now undertaken. In this paper, we will simply lend from the method proposed by Yang (1998), which substitutes the direct measure of estimated OD matrix quality by some related indices.

Literature Review and Research Objectives

In review of location optimization related literature, it appears that most models for the optimal facility location are formulated to intercept a maximum number of customer flows (Larson, 1981; Hodgson, 1990; Berman, 1992). For instance, to relax the assumption of knowing all path flows in their early model, Berman et al. (1995) formulated an average-reward Markov decision process that requires only the knowledge of turning fractions at each node and all originating demands.

As for location selections related to OD estimation, most early pioneering studies on this regard, such as random selection method and major link selection method (Han, 1983), are intuitive in nature. Logie and Hynd (1990) in later years proposed the OD coverage method, and Lam and Lo (1990) in the same period also proposed some heuristic procedures to identify the sequence for network link selection.

Among existing studies, one of the most effective methods was proposed by Yang and Zhou (1998), who, based on the concept of maximal possible relative error (MPRE) and some numerical results, designed four rules to locate traffic counting detectors (i.e., OD covering rule, maximal flow fraction rule, maximal flow-intercepting rule, and link independence rule). Using these rules instead of defining the quality of OD estimation directly, they formulated the following two methods: (1) A LP model to maximize the net traffic flows intercepted while keeping all OD pairs covered, and (2) an application of Berman's model (1995) to formulate the entire problem as an average-reward Markov Decision Process with the objective of maximizing the net captured flows.

Grounded on Yang and Zhou's results (1998), Yim and Lam (1998) later formulated one LP model to maximize both the net and the total captured flows. An approximate heuristic algorithm was also proposed as an alternative approach in their work. To distinguish, net flows here means the sum of link flows after excluding double counting effects, while total flows means the sum of link flows including double counting effects.

Despite the promising properties of Yang and Zhou's method, one of the most advanced studies on this research issue, there are some embedded limitations to be overcome. For example, Yang and Zhou's models intend to maximize the net traffic flows intercepted by traffic counting detectors. Such an objective function has the inherent limitation that the net flow intercepted has an apparent upper bound, namely the total flow on the network F.

Thus, when the number of locations in the constraint l_0 is larger than the "desirable number" $|L_{opt}|$, the minimal number of links to cover the total flow F, the model cannot guide the selection of other detector locations after the objective function has reached the upper bound. This indicates that the remaining links may be selected arbitrarily and the resulting quality of the estimated OD matrix will be degraded accordingly.

For example, as showed in Figure 1, Yang and Zhou's heuristic gives $|L_{opt}| = 1$ by selecting link 3 and the objective function reaches its upper bound 210. Consider $l_0 = 2$ and the true / target OD given in Table 1. The sum of squared errors for OD estimation by selecting another link along with link 3, computed with the OLS model, is given in Table 2, which clearly indicates that the resulting quality of estimated OD matrix varies significantly with the selected links.

To circum the above limitation, Yim and Lam (1998) established the rule of maximal total captured flow based on Yang and Zhou's previous analysis. This rule indicates that more total flows intercepted will lead to higher estimation reliability when the same amount of net flows is intercepted. However, this rule does not always hold. Sometimes more gross flows can lead to lower estimation reliability when the same amount of net flows is intercepted.

A counter example is given to facilitate the discussion. Based on the same true OD matrix and target OD matrix in Table 1, the sum of squared errors for OD estimation computed by the OLS model and the resulting net and gross flow intercepted are given in Table 3. As shown in the Table 3, although all the groups of links intercept an equal amount of net flows, the estimation quality does not exhibit a correlation with the gross flows intercepted. For instance, set $\{3,1\}$ intercepts 38.1% more gross flows than set $\{1,2\}$, but both sets have about the same estimation quality. Similarly, set $\{3,2\}$ intercepts 25.9% more gross flows than set $\{3,4\}$, but has a lower estimation quality. Hence, introducing gross flows into the original model may not improve the estimation results.

Besides, Yang and Zhou (1998) proposed the following heuristic greedy algorithm to determine the desirable number and locations of counting detectors. Defining the set of all paths as R, their algorithm assumes the knowledge of all path flows $\{f_r, r \in R\}$ and a coverage matrix $\boldsymbol{B} = [b_{ra}]$, where $b_{ra} = 1$ if link a is on path r, and 0 otherwise. Thus link flows can be expressed as

$$v_a = \sum_{r \in R} b_{ra} f_r = \mathbf{f}\boldsymbol{B}_a$$

Where $\mathbf{f}$ is a row vector of path flows and $\boldsymbol{B}_a$ is the ath column vector of coverage matrix $\boldsymbol{B}$. Links are then selected in a descending order of their carried flows so as to maximize the total flows intercepted. Besides, to exclude double counting effects and to keep link flows independent, their algorithm revises the path flow vector $\mathbf{f}$ after selecting a link by changing all the path flows intercepted by this link to zero. Accordingly, flows on all of the remaining links are also revised.

If there is no constraint on the total number of counting locations, this algorithm will yield a location vector $\boldsymbol{L}_{opt}$ that can observe all flows traveling through the network, which is the maximal net flow that can possibly be intercepted. This optimal location vector $\boldsymbol{L}_{opt}$ can also cover all OD pairs and keep link flows independent of each other.

The authors also claimed that this algorithm could be used as a heuristic to solve the problem with budget constraints. But when the number of counting locations l_0 is less than the desirable number $|L_{opt}|$, which is obtained without budget constraints, the result may be somewhat undesirable when employing the OD covering rule.

More specifically, this heuristic algorithm tries to maximize the net traffic flows intercepted by accumulating path flows. In the selection of links, considerations are only given to the amount of flows carried by paths passing this link, which contains no information on the OD pair correlated. Thus, when the number of counting locations is less than the desired number, some path flows may not be accumulated even if they are the only source for OD information between certain OD pairs. This implies that the estimation result for these omitted OD pairs may have an unbounded range of errors.

A simple example is given in Figure 2 where $l_0 = 2$. Apparently, the algorithm will select link (path) 1 and 2 and leave OD pair OD_2 uncovered. The result has violated the OD covering rule.

An Enhancement Algorithm

In view of above-mentioned limitations in the existing literature, this study, grounded on the core concept by Yang (1998), has proposed an efficient algorithm to contend with the critical detector location issue.

To satisfy the OD covering rule, the core of our proposed idea is: (1) first to check the minimal number of links that can cover all OD pairs l_{min} so as to recognize a too low budget constraint l_0; and then (2) to check the number of OD pairs to be covered when selecting links for locating detectors.

More specifically, we define the total number of OD pairs as $|W|$, and the number of OD pairs already covered after selecting l links as $|W|_l$. If $|W| - |W|_l < l_0 - l$, namely the number of OD pairs to be covered is less than the number of locations to be selected, then we shall select the $(l+1)th$ links according to Yang's algorithm. Otherwise, check if some new OD pairs are covered by the newly selected link. If this link provides no information on new OD pair, it shall be denied, and the next link shall be selected and checked in the same manner. If $l_{min} < l_0$ but no solution satisfying OD covering rule is found, the current solution shall be revised by deleting the selected link covering the least number of new OD pairs and adding the unselected link covering the most number of new OD pairs.

To solve the scenario in which the net flow intercepted has reached its upper bound but the number of selected links remains below the budget constraint l_0, the basic idea is: (1) first to check the minimal number of links that can intercept all net flow $|L_{opt}|$; and (2) if $|L_{opt}| < l_0$, then to include all links in L_{opt} and to select additional links according the maximal flow fraction rule.

The logic flow of the proposed heuristic is described as follows:

Step 0: *Initialization.*

Build path flows vector $\mathbf{f}$ and coverage matrix $\boldsymbol{B}$.

Set $|W|$ as the total number of OD pairs, counter $l = 0$, and $|W|_l = 0$.

Step 1: *Basic Judgment.*

Solve for the minimal number of links covering all OD pairs l_{min}.

Solve for the minimal set of links intercepting all net flow L_{opt}.

If $l_{min} > l_0$, select the first l_0 links from L_{opt} and indicate "some OD pairs may not be covered. If $|L_{opt}| > l_0$, go to step 2. Otherwise go to step 8.

Step 2: *OD Covering Rule Checking.*

If $|W| - |W|_l < l_0 - l$: go to Step 3. Else: go to Step 4.

Step 3: *Basic Selection.*

Accept link $b : v_b = \max_a (v_a = \mathbf{fB}_a)$. $l = l + 1$, compute $|W|_l$. Go to Step 5.

Step 4: *Selection Based on OD Covering Rule.*

Step 4_1: Set the label of all links as "non-denied".

Step 4_2: Select link $b : v_b = \max_{label(a)=non-denied} (v_a = \mathbf{fB}_a)$. Set $l = l + 1$

Step 4_3: Compute $|W|_l$. If $|W|_l > |W|_{l-1}$: Accept link b and go to step 5.

Else: Deny b. Set $label(b) = denied$, $l = l - 1$ and go to step 4_2.

Step 5: *Convergence Test.*

If $l = l_0$: go to step 7. Else: go to step 6.

Step 6: U*pdate.*

Modify $\mathbf{f}$ by changing all path flows intercepted by the selected link to zero. Go to step 2.

Step 7: *OD Covering Rule Re-checking.*

If $|W| - |W|_{l0} > 0$, then

Step 7_1: Delete the selected link covering the least number of new OD pairs

Step 7_2: Add the unselected link covering the most number of new OD pairs

Step 7_3: Go to step 7;

If $|W| - |W|_{l0} = 0$, stop.

Step 8: *Additional Link Selection.*

Choose L_{opt} as part of the solution and select additional links according to the maximal flow fraction rule.

Stop.

Numerical Experiments

To evaluate the proposed heuristic algorithm, we have designed a set of numerical experiments based on the network shown in Figure 3. There are a total of 63 links in the network, including 61 two-way links and 2 one-way links. To facilitate the computation, each two-way link is changed to two one-way links in the database according to the node sequence.

There are five origin-destination nodes indicated with black circles in Figure 3, thus the OD matrix is a 5×5 matrix. Neglecting trips in the same zone, we have randomly generated trips between 200 and 600 for the remaining 20 pairs of OD (see Table 4).

Based on free flow travel time on links, four shortest paths are selected between each OD pair. Path flows are computed with the following equation.

$$f_k^{rs} = \frac{tt_k^{rs}}{\sum_{i=1}^{4} tt_i^{rs}} \times q_{rs}, \qquad \forall r = 1,2,\ldots,5, \qquad s = 1,2,\ldots,5, \qquad k = 1,2,\ldots,4,$$

Three heuristic algorithms are programmed in Visual Basic 6.0. The first algorithm is Yang and Zhou's heuristic without considering budget constraints. The second algorithm is Yang and Zhou's heuristic, but incorporated with the budget constraints. The third algorithm is our proposed heuristic that includes the budget constraints and the OD covering rule.

The optimal set of detector locations without considering budget constraints are given in Table 5. A total of 12 locations have been selected, which intercept a total net flow of 7900, i.e., the total OD flows in the network. Based on the computational results, a minimal number of five links are needed to cover all OD pairs in the network. Thus, we have $|L_{opt}| = 12$ and $l_{min} = 5$.

As indicated above, the proposed heuristic algorithm should be able to handle three different situations, namely $l_0 \geq |L_{opt}|$, $l_{min} \leq l_0 < |L_{opt}|$, and $l_0 < l_{min}$. Thus, by setting the upper bound for detector locations $l_0 = 15, 14, \ldots$ respectively, we test the new heuristic algorithm and Yang's heuristic algorithm under budget constraints. Five typical cases of the numerical results are shown in Table 6-1 to Table 6-5.

In Case 1, the budget constraint $l_0 = 15$ is larger than the minimal number of links needed to intercept all net flow, $|L_{opt}| = 12$. Our new algorithm will directly include all links in L_{opt}, and choose additional links based on the Maximal Flow Fraction Rule proposed by Yang and Zhou (1998). According to the theoretical analysis by Yang and Zhou, this kind of selection is reasonable from the perspective of Maximal Possible Relative Error (MPRE) in the OD estimation. However, since the path flow vector $\mathbf{f}$ in Yang's algorithm will contain solely zero elements after the selection of L_{opt}, additional links can only be selected randomly, and consequently the quality of OD estimation will be degraded.

In the remaining of cases $l_0 < |L_{opt}|$, Yang's algorithm will choose the first l_0 links in the set of L_{opt} as its solution.

In Case 2, the budget constraint is relatively high and the solution obtained by Yang's algorithm also satisfies OD covering rule. The result shows an equivalent effect of the proposed algorithm in this case.

In Case 3, the budget constraint goes down to 8. The solution obtained by Yang's algorithm, as shown in Table 6-3, can only cover 18 OD pairs. The solution with our proposed heuristic can still cover all OD pairs, but with slightly lower net flow intercepted (i.e., 6765 vs. 6954).

In Case 4, when the budget constraint further goes down to 5, our proposed algorithm will activate its final revision step in the *OD Covering Rule Re-checking procedures*, and yield the coverage of 20 OD pairs, that is five pairs more than those would be covered with Yang's algorithm.

In Case 5, when the budget constraint l_0 is less than the minimal number of links to cover all OD pairs l_{min}, our proposed algorithm will provide the same solution as Yang's algorithm and also give a message to indicate that the budget constraint is too low to cover all OD pairs.

Note that the numerical results in the above five cases seem to support the effectiveness of our proposed heuristic algorithm in satisfying the OD covering rule, especially under insufficient budget constraints. Also note that since both heuristic algorithms, to some extent, belong to the category of greedy algorithm, neither can guarantee the global maximum in their net flow intercepted when $l_0 < |L_{opt}|$.

Conclusions and Future Research Issues

This paper has discussed critical issues associated with optimizing detector locations for OD matrix estimation, and proposed an enhanced heuristic algorithm that can effectively take into account the budget constraints and the OD coverage. Although much remains to be improved in contending with the need in real-world applications, the numerical results have revealed a promising property of our proposed algorithm.

One of the most critical issues remains to be researched is how to define the quality of OD matrix estimation. Maximal Net Flow Interception Rule and Maximal

Flow Fraction Rule are adopted here to define the estimation quality indirectly. However, these rules are far from sufficient, and other factors such as network characteristics may also play an important role. We are now trying to directly include the uncertainty of OD estimation problem into the selection of detector locations. Our future research work will be focused on a series of numerical experiments specially designed to test: (1) the robustness of current rules on various types of network, and (2) the sensibility of the role played by network characteristics and prior OD information.

Acknowledgements

The authors would like to appreciate some valuable comments from Dr. Mahmassani on the early version of the work.

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List of Tables

List of Figures

Table 1. True and Target OD Matrices

True OD			
	D_1	D_2	D_3
O_1	20	30	30
O_2	40	50	40

Target OD			
	D_1	D_2	D_3
O_1	15	20	20
O_2	30	40	30

Table 2. Sum of Squared Errors for OD Estimation by OLS

Links Selected	{3}	{3,1}	{3,2}	{3,4}	{3,5}	{3,6}
Sum of Squared Errors	20.83	16.67	16.67	12.50	18.75	18.75

Table 3. Sum of Squared Errors for OD Estimation with Different Flow Information

Links Selected	{1,2}	{3}	{3,1}	{3,2}	{3,4}	{3,5}	{3,6}
Net Flow Intercepted	210	210	210	210	210	210	210
Gross Flow Intercepted	210	210	290	340	270	290	280
Sum of Squared Errors	16.67	20.83	16.67	16.67	12.50	18.75	18.75

Table 4. True OD Matrices

O \ D	1	2	3	4	5	$\sum$
1	0	330	550	520	330	1730
2	370	0	240	440	230	1280
3	320	250	0	520	280	1370
4	370	270	500	0	370	1510
5	410	480	570	550	0	2010
$\sum$	1470	1330	1860	2030	1210	7900

Table 5. Optimal Detector Locations without Budget Constraints

Link ID	9	109	14	108	118	102
Net Flow Intercepted	2010.0	1210.0	807.1	705.5	646.0	570.2
Link ID	34	101	42	149	2	105
Net Flow Intercepted	514.4	489.8	352.8	336.0	134.1	124.0

Table 6-1. Optimal Detector Locations under Budget Constraints: *Case 1*

<table>
<tr><td rowspan="18">$l_0 = 15$</td><td colspan="3">New Heuristic</td><td colspan="3">Yang's Heuristic</td></tr>
<tr><td>Link ID</td><td>Net Flow Intercepted</td><td>OD Pairs Covered</td><td>Link ID</td><td>Net Flow Intercepted</td><td>OD Pairs Covered</td></tr>
<tr><td>9</td><td>2010.0</td><td>4</td><td>9</td><td>2010.0</td><td>4</td></tr>
<tr><td>109</td><td>1210.0</td><td>8</td><td>109</td><td>1210.0</td><td>8</td></tr>
<tr><td>14</td><td>807.1</td><td>11</td><td>14</td><td>807.1</td><td>11</td></tr>
<tr><td>108</td><td>706.0</td><td>13</td><td>108</td><td>706.0</td><td>13</td></tr>
<tr><td>118</td><td>646.0</td><td>15</td><td>118</td><td>646.0</td><td>15</td></tr>
<tr><td>102</td><td>570.2</td><td>18</td><td>102</td><td>570.2</td><td>18</td></tr>
<tr><td>34</td><td>514.5</td><td>18</td><td>34</td><td>514.5</td><td>18</td></tr>
<tr><td>101</td><td>489.8</td><td>18</td><td>101</td><td>489.8</td><td>18</td></tr>
<tr><td>42</td><td>352.8</td><td>20</td><td>42</td><td>352.8</td><td>20</td></tr>
<tr><td>149</td><td>336.0</td><td>20</td><td>149</td><td>336.0</td><td>20</td></tr>
<tr><td>2</td><td>134.1</td><td>20</td><td>2</td><td>134.1</td><td>20</td></tr>
<tr><td>105</td><td>124.0</td><td>20</td><td>105</td><td>124.0</td><td>20</td></tr>
<tr><td>40</td><td>0</td><td>20</td><td colspan="3" rowspan="4">Other links will be randomly selected</td></tr>
<tr><td>122</td><td>0</td><td>20</td></tr>
<tr><td>22</td><td>0</td><td>20</td></tr>
<tr><td>$\sum$</td><td>7900</td><td>20</td></tr>
</table>

Table 6-2. Optimal Detector Locations under Budget Constraints: *Case 2*

	New Heuristic			*Yang's Heuristic*		
	Link ID	Net Flow Intercepted	OD Pairs Covered	Link ID	Net Flow Intercepted	OD Pairs Covered
$l_0 = 11$	9	2010.0	4	9	2010.0	4
	109	1210.0	8	109	1210.0	8
	14	807.1	11	14	807.1	11
	108	706.0	13	108	706.0	13
	118	646.0	15	118	646.0	15
	102	570.2	18	102	570.2	18
	34	514.5	18	34	514.5	18
	101	489.8	18	101	489.8	18
	42	352.8	20	42	352.8	20
	149	336.0	20	149	336.0	20
	2	134.1	20	2	134.1	20
	$\sum$	**7776.0**	**20**	$\sum$	**7776.0**	**20**

Table 6-3. Optimal Detector Locations under Budget Constraints: *Case 3*

	New Heuristic			*Yang's Heuristic*		
	Link ID	Net Flow Intercepted	OD Pairs Covered	Link ID	Net Flow Intercepted	OD Pairs Covered
$l_0 = 8$	9	2010.0	4	9	2010.0	4
	109	1210.0	8	109	1210.0	8
	14	807.1	11	14	807.1	11
	108	706.0	13	108	706.0	13
	118	646.0	15	118	646.0	15
	102	570.2	18	102	570.2	18
	17	494.3	19	34	514.5	18
	149	322.1	20	101	489.8	18
	$\sum$	**6765.3**	**20**	$\sum$	**6953.6**	**18**

Table 6-4. Optimal Detector Locations under Budget Constraints: *Case 4*

	New Heuristic			*Yang's Heuristic*		
	Link ID	Net Flow Intercepted	OD Pairs Covered	Link ID	Net Flow Intercepted	OD Pairs Covered
$l_0 = 5$	149	897.5	5	9	2010.0	4
	123	1210.2	9	109	1210.0	8
	52	1010.2	13	14	807.1	11
	136	823.4	16	108	706.0	13
	105	736.4	20	118	646.0	15
	$\sum$	**4677.7**	**20**	$\sum$	**5949.3**	**15**

TABLE 6-5. Optimal Detector Locations under Budget Constraints: *Case 5*

	New Heuristic			*Yang's Heuristic*		
	Link ID	Net Flow Intercepted	OD Pairs Covered	Link ID	Net Flow Intercepted	OD Pairs Covered
$l_0 = 3$	9	2010.0	4	9	2010.0	4
	109	1210.0	8	109	1210.0	8
	14	807.1	11	14	807.1	11
	$\sum$	**4027.1**	**11**	$\sum$	**4027.1**	**11**
	Warning Information "cannot cover all OD pairs!"			**No Indication**		

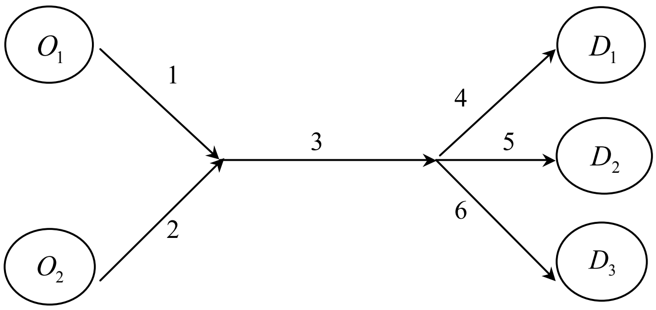
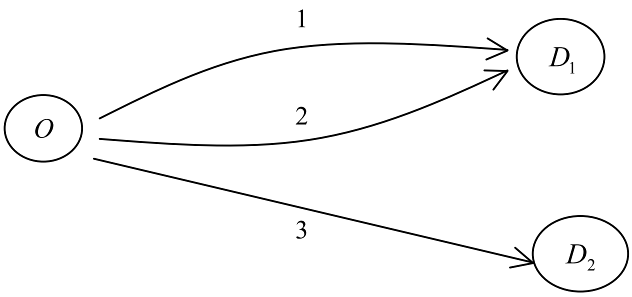


Figure 1. An Example When Objective Function Is Maximal With One Detector



$$\mathbf{f}^{T} = \begin{bmatrix} 5 \\ 4 \\ 3 \end{bmatrix} \qquad \boldsymbol{B} = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix}$$

Figure 2. An Example When the Algorithm Violates OD Covering Rule

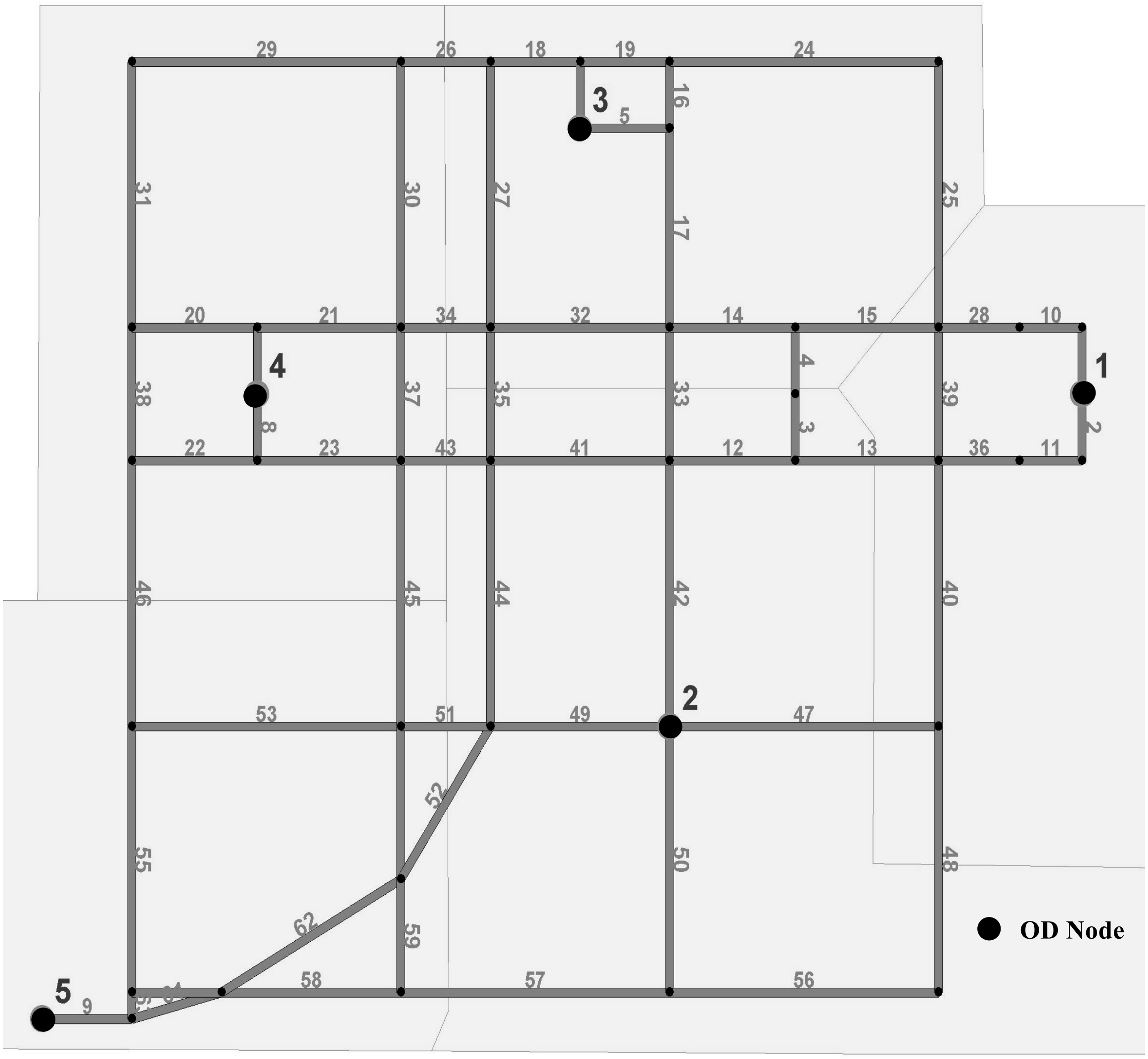


Figure 3. Network Used in Numerical Experiments