Robust Model for Estimating Freeway Dynamic Origin–Destination Matrix

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This study presents a robust model for estimating the dynamic freeway origin–destination matrix with a measurable time series of ramp and mainline flows. The proposed model captures the speed variance among vehicles having the same departure time, origin, and destination with an embedded travel time distribution function that results in a substantial reduction in model parameters. With the developed solution algorithm, the proposed model offers the potential use in a network of realistic size such as the I-95 freeway corridor between the Maryland I-695 and I-495 beltways. Extensive numerical analyses with respect to the sensitivity of both input measurement errors and the selection of initial parameters have revealed that the proposed model is sufficiently robust for real-world applications.

Dynamic origin–destination (O-D) matrices are essential input information for a variety of traffic control applications, such as real-time route guidance and dynamic traffic assignment. Because the actual number of variables to be estimated for either a static or a dynamic system is always far greater than the available information, transportation researchers over the past two decades have used various methods to contend with this difficult issue. In reviewing the related literature, it is noticeable that recent studies for dynamic O-D estimation can be classified into two main categories: assignment- and non-assignment-based approaches. The former category of approaches uses an assumption that a reliable prior O-D set and a dynamic traffic assignment model that predicts route choice behavior are available (*1–10*). Considering the practical difficulty in having a reliable prior O-D, some researchers have devoted themselves to developing various estimation approaches that can use only the time series of available volume counts and thus reduce the dependency of the prior O-D information (*11–23*). This study follows the research line of non-assignment-based methods and intends to estimate the dynamic freeway O-D based mainly on all observable link and ramp flow rates.

Consider a typical freeway corridor of N segments ranging from 0 to $N - 1$ with link count information as indicated in Figure 1, where detectors are placed at on-ramps, off-ramps, and mainline links. The information readily available for estimation of its time-dependent O-D flow proportion or dynamic O-D distribution is the time series of entering flow [$q_i(k)$], exiting flow [$y_i(k)$], and mainline flow [$U_l(k)$].

Let $b_{ij}(k)$ denote the proportion of vehicles entering from origin i to destination j during time interval k. By definition, it shall be subjected to the following two constraints:

$$0 \le b_{ij}(k) \le 1 \qquad 0 \le i \le j \le N \tag{1}$$

$$\sum_{j=i+1}^{N} b_{ij}(k) = 1 \qquad i = 0, 1, \ldots, N-1 \tag{2}$$

With the assumption that the trip time is negligible, the relations between entry and exiting flows can be formulated as follows (*12*):

$$y_j(k) = \sum_{i=0}^{j-1} b_{ij}(k) q_i(k) \qquad j = 1, 2, \ldots, N \tag{3}$$

The number of unknown variables for the example freeway as indicated in Figure 1 is $N \times (N+1)/2$, and the number of equations for Equations 3 is N. Obviously, when N is >1, the model is underdetermined, as there are more unknown parameters [$b_{ij}(k)$] than the system equations.

To overcome the underdetermined nature, some studies assumed that certain relations exist between O-D patterns during successive time intervals. The entire model can then be reformulated and solved with statistical methods such as generalized least squares and constrained least squares (*11–16*).

Most such models, based on input/output flow, use the assumption that travel time between origins and destinations is either constant or negligible. However, when the travel time is significantly long so as to affect the input and output flow relationships, Equation 3 is no longer valid, and the travel time factors must be explicitly captured in the dynamic formulations. To contend with this issue, Bell (*21*) first modeled the travel time factor in the presence of platoon dispersion. In analyzing the turning movements at intersections, Bell's study assumed that travel time needed for vehicles to traverse the intersection does not exceed the length of one control time interval. Using the platoon dispersion relation, he reformulated Equation 1 as the following linear model:

$$y_j(k) = \left(1 - \alpha_j\right) y_j(k-1) + \alpha_j \; q^T(k) \; b_j(k) \tag{4}$$

where the additional smoothing parameter α_j ($0 \le \alpha_{ij} \le$) also needs to be estimated. However, by doing so, the number of unknown variables has been increased to $N \times (N+3)/2$, and the number of system equations remains N.

Bell (*22*) further proposed an extended linear model dealing with freely distributed travel times. The O-D proportion parameters are decomposed with the travel time distributions

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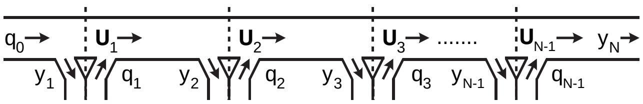

FIGURE 1 Typical freeway corridor.

$$y_j(k) = \sum_{m=0}^{M} \sum_{i=0}^{j=1} q_i(k-m) b_{ijm} \qquad j = 1, 2, \ldots, N \tag{5}$$

where b_{ijm} denotes the proportion of trips from entrance i destined to exit j with a travel time of m intervals, and m is the number of time intervals required for vehicles from entrance i to exit j.

Equation 5 offers a more realistic formulation as the travel time for an O-D pair may not be within the length of one unit time interval. However, it cannot be effective if travel time spans more than two time intervals. This is primarily due to the poor observability of the system equations that involves relatively too many parameters b_{ijm}. In addition, the assumption that the O-D flow proportion b_{ijm} remains constant over the time period of interest may not be realistic.

Chang and Wu (*23*) proposed a freeway O-D estimation model by using information from both mainline flow counts [$U_l(k)$] and ramp flow measurements [$q_i(k)$ and $y_j(k)$] to construct a set of dynamic equations. To further capture the relation between O-D flow proportions and traffic counts, they proposed a set of new variables, $\theta^-_{ij}(k)$ and $\theta^+_{ij}(k)$, to represent the fraction of $q_i(k-m) \cdot b_{ij}(k-m)$ trips that arrive at off-ramp j during time interval k. In their model, the number of unknown variables becomes $(M+1) \times N \times (N+1)/2$ and the number of system equations increases to $2N-1$. To improve the operational efficiency, they also proposed an algorithm that aims to estimate an average $b_{ij}(k)$ for some consecutive time intervals instead of solving an O-D flow distribution matrix for each time interval. The number of unknown variables under such a refined formulation reduces to $3N \times (N+1)/2$. Their formulations are based on the assumption that the speeds of vehicles entering the freeway at the same time interval are distributed in a small range.

In brief, to advance existing models for real-world application, one needs to overcome the following three critical issues: the first is that the system equations for O-D estimation only from traffic counts are clearly underdetermined as the number of equations is always far less than the O-D pairs. The second is that a developed model shall have the capability of formulating a large-scale freeway network. The last issue is to release the commonly used assumption that all entries and exiting flow counts are available, or a reliable set of prior O-D for model calibration exists. In reality, such information may be neither complete nor accurate at the desirable level. Focusing on these three issues, the study first proposes a model with an embedded function that allows the travel times to exceed the unit control interval and vary over a wide range. The applicability of the proposed model to a large freeway network (i.e., the I-95 freeway corridor) is also investigated. Finally, the robustness of the estimation results under potential measurement errors is also evaluated with numerical experiments.

The remainder of this paper is organized as follows: the basic relations between the time-dependent O-D flows and the time-series traffic measurements in a freeway corridor formulated with a nonlinear dynamic system model are illustrated in the next section; the solution algorithm developed with the extended Kalman filtering method is presented in the third section; simulation experiments and sensitivity analyses for model stability evaluation are reported in the fourth section; and conclusions and further enhancements are summarized in the last section.

MODEL FORMULATION

Consider a freeway corridor of N segments from 0 to $N-1$ as indicated in Figure 1, the set of variables used in modeling its dynamic traffic flow and O-D relations:

- $q_0(k)$ = number of vehicles entering the upstream boundary of the freeway section during time interval *k;*
- $q_i(k)$ = number of vehicle trips entering freeway from on-ramp i during time interval k, $i = 1, 2, \ldots, N-1$;
- $y_j(k)$ = number of vehicle trips leaving the freeway from off-ramp j during time interval k, $j = 1, 2, \ldots, N-1$;
- $y_n(k)$ = mainline volume at the downstream end of the freeway section during time interval *k;*
- $U_i(k)$ = number of vehicles crossing the upstream boundary of segment i during time interval k, $i = 1, 2, \ldots, N-1$;
- $T_{ij}(k)$ = number of vehicles entering the freeway from on-ramp i during time interval k that are destined to off-ramp j (i.e., the time-dependent O-D flow), where $0 \le I < j \le N$;
- t_0 = length of one unit time interval;
- $t_{ij}(k)$ = average travel time of vehicles from on-ramp i to off-ramp j departing during time interval *k;*
- $\sigma_{ij}(k)$ = standard deviation of the travel time for vehicles from on-ramp i to off-ramp j departing during time interval *k;*
- $b_{ij}(k)$ = proportion of $q_i(k)$ heading toward destination node j during time interval *k;* and
- $\theta^m_{ij}(k)$ = fraction of $T_{ij}(k-m)$ trips that arrive at off-ramp j during time interval k.

With the preceding definitions, one can establish the following relations based on Figure 1:

$$q_i(k) = \sum_{j=i+1}^{N} T_{ij}(k) \qquad i = 0, 1, \ldots, N-1 \tag{6}$$

$$T_{ij}(k) = q_i(k) b_{ij}(k) \qquad 0 \le i < j \le N \tag{7}$$

Equations 6 and 7 are subjected to Constraints 1 and 2 discussed previously.

In view of the speed variation among drivers, it is reasonable to assume that the departure times for vehicles from node i arriving at node j during time interval k are distributed among time intervals k, $k-1, \ldots,$ and $k-M$, where M is the maximum number of intervals required for vehicles to traverse the entire freeway section. The exit traffic volume [$y_j(k)$] can thus be stated as follows:

$$\begin{aligned} y_j(k) &= \sum_{m=0}^{M} \sum_{i=0}^{j=1} T_{ij}(k-m)\theta^m_{ij}(k) \\ &= \sum_{m=0}^{M} \sum_{i=0}^{j=1} q_i(k-m)\theta^m_{ij}(k) b_{ij}(k-m) \qquad j = 1, 2, \ldots, N \end{aligned} \tag{8}$$

where $\theta^m_{ij}(k)$, a set of new time-dependent parameters, or state variables, shall satisfy the following relations:

$$0 \le \theta^m_{ij} \le 1 \qquad 0 \le i \le j \le N \qquad m = 0, 1, \ldots, M \tag{9}$$

$$\sum_{m=0}^{M}\theta_{ij}^{m}(k+m)=1 \quad 0\le i<j\le N \tag{10}$$

As discussed in the work by Chang and Wu (*23*), Equation 8 is sufficient for capturing the dynamic relationships between O-D patterns and link flows if the freeway is not congested and traffic flow is stable. Otherwise, the time-varying traffic volume $U_l(k)$ cannot be determined simply with the entrance and exit flow data $q_i(k)$ and $y_j(k)$. Hence, the measurements of $\{U_l(k)\}$ may actually provide additional valuable information for estimation. A new set of constraints that uses the mainline traffic volume $U_l(k)$ is given as follows:

$$U_{\ell}(k)=\sum_{m=0}^{M}\sum_{i=0}^{\ell=1}\sum_{j=\ell+1}^{N}q_i(k-m)\theta_{i\ell}^{m}(k)b_{ij}(k-m)+q_{\ell}(k)$$

$$U_{\ell}(k)-q_{\ell}(k)=\sum_{m=0}^{M}\sum_{i=0}^{\ell=1}\sum_{j=\ell+1}^{N}q_i(k-m)\theta_{i\ell}^{m}(k)b_{ij}(k-m) \tag{11}$$

where $\ell = 1, 2, \ldots, N-1$.

However, it is noticeable that the system formulation contains a large number of state variables—that is, $b_{ij}(k)$ and $\theta_{ij}^{m}(k)$. The number of these unknown parameters increases with the required M value. As such, some more information and refinement are necessary to ensure that this proposed model is computationally efficient and tractable.

To do so, one can assume that the travel times of drivers departing from i during time interval k to j follow a normal distribution, $N[\mu_{ij}(k), \sigma_{ij}(k)]$, as indicated in Figure 2,

where

$\mu_{ij}(k) = t_{ij}(k)$ = average travel time of vehicles from i to j departing during interval k,

$\sigma_{ij}(k)$ = standard deviation of the travel time of vehicles from i to j departing during interval k, and

$\rho_{ij}^{m}(k)$ = fraction of $T_{ij}(k-m)$ trips from on-ramp i during interval k that takes m time intervals to j.

The use of normal distribution to approximate the travel time distribution of vehicles with the same O-D has been reported in the literature (*22, 24*), and this assumption was also supported by Grace and Potts (*25*). Furthermore, Seddon (*26*) has examined the theoretical basis for the recurrence model and found that it corresponds to Pacey's (*27*) diffusion model of platoon dispersion when the normal distribution for vehicle speeds is replaced with the shifted geometric distribution for travel times.

As indicated in Figure 3, because the travel time for an O-D pair departing during the same time interval follows a normal distribution,

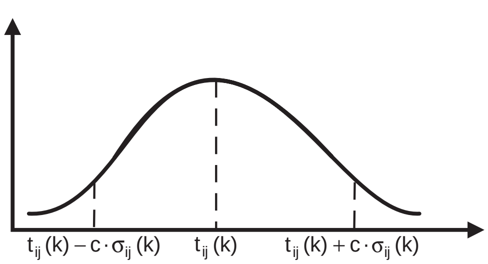

FIGURE 2 Assumed distribution of travel times for drivers from *i* during interval *k* to *j*.

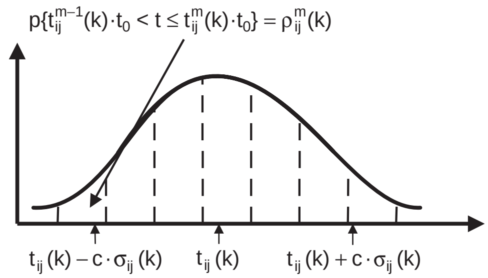

FIGURE 3 Probability of travel time distribution.

$\rho_{ij}^{m}(k)$ can be replaced with a cumulative density function within a time interval, m, as follows:

$$\rho_{ij}^{m}(k) = \int_{m\cdot t_0}^{(m+1)\cdot t_0} f(x)dx \tag{12}$$

$$\sum_{m=0}^{M}\rho_{ij}^{m}(k-m) = 1 \tag{13}$$

where $0 \le \rho_{ij}^{m}(k) \le 1$, $0 < i \le j \le N$, and $m = 0, 1, \ldots, M$.

In addition to the use of a normal distribution to represent the variation of travel time, one can also estimate the average $\bar{b}_{ij}(k)$ for consecutive intervals, instead of solving an O-D flow distribution matrix for each small interval (*23*). Thus, all the $b_{ij}(\cdot)$ terms in Equations 8 and 11 can be replaced with $\bar{b}_{ij}(k)$

$$\begin{aligned} y_j(k) &= \sum_{m=0}^{M}\sum_{i=0}^{j=1}\left[q_i(k-m)\rho_{ij}^{m}(k)\right]\cdot\bar{b}_{ij}(k) \\ &= \sum_{m=0}^{M}\sum_{i=0}^{j-1}\left[q_i(k-m)\cdot\int_{m\cdot t_0}^{(m+1)\cdot t_0}f(x)dx\right]\cdot\bar{b}_{ij}(k) \end{aligned} \tag{14}$$

$$\begin{aligned} U_{\ell}(k)-q_{\ell}(k) &= \sum_{m=0}^{M}\sum_{i=0}^{\ell-1}\sum_{j=\ell+1}^{N}\left[q_i(k-m)\rho_{ij}^{m}(k)\right]\cdot\bar{b}_{ij}(k) \\ &= \sum_{m=0}^{M}\sum_{i=0}^{\ell-1}\sum_{j=\ell+1}^{N}\left[q_i\left(k-m\cdot\int_{m\cdot t_0}^{(m+1)\cdot t_0}f(x)dx\right)\right]\cdot\bar{b}_{ij}(k) \end{aligned} \tag{15}$$

With the preceding reformulations, the average travel time for each O-D pair can be measured from the surveillance system, and the unknown sets of variables are O-D proportions $\bar{b}_{ij}(k)$ and standard deviations $\sigma_{ij}(k)$.

SOLUTION ALGORITHM

The number of unknown parameters under the revised formulations is reduced to $2N \times (N+1)/2$, compared with the model proposed by Chang and Wu (*23*). However, because of the nonlinear nature of the formulations and concerns about computing efficiency, this study has used the sequential extended Kalman filtering algorithm (*28*) and the Gumbel distribution (an approximation of the normal distribution) to develop the solution algorithm. A step-by-step description of the algorithm for estimating the parameters $b_{ij}(k)$ and $\sigma_{ij}(k)$ is presented as follows:

Step 0. Initialize

- Link length L_i, $i = 0, 1, \ldots, N-1$
- Length of each time interval t_0, and the maximal number of intervals required to traverse the entire section M
- Initial input mean speeds, $V_i(m)$, $m = -M, -M+1, \ldots, 0$
- Initial input flows, $q_i(m)$, $m = -M, -M+1, \ldots, 0$
- Initial travel times, $t_{ij}(m) = L_i / V_i(m) + \cdots + L_{j-1}/V_{j-1}(m)$, $m = -M, -M+1, \ldots, 0$
- $\mathrm{Var}[e(k)] = \mathrm{diag}[r_1, r_2, \ldots, r_{2N-1}]$

$$\begin{bmatrix} b(0) \\ \sigma(0) \end{bmatrix} = E\begin{bmatrix} b(0) \\ \sigma(0) \end{bmatrix}, P_0 = \mathrm{Var}\begin{bmatrix} b(0) \\ \sigma(0) \end{bmatrix}$$

where

$$b(k) = [b_{ij}(k)] = \begin{bmatrix} b_{01}(k) & b_{02}(k) & \ldots & b_{0,N}(k) \\ b_{11}(k) & & & \\ \vdots & & & \\ b_{N-1,1}(k) & & & b_{N-1,N}(k) \end{bmatrix}$$

$$\sigma(k) = [\sigma_{01}(k), \sigma_{02}(k), \ldots, \sigma_{0N}(k), \sigma_{11}(k), \ldots, \sigma_{N-1,N}(k)]^T$$

Step 1. Compute travel time (mean value)

$$u_{ij}(k) = t_{ij}(k)$$

Step 2. Compute linearized transformation matrix

- $H^{k-1} = [H^{k-1}_{rs}]_{(2N-1)\times N(N+1)/2}$

$$H^k_{j,Ni+j-i(i+1)/2} = \sum_{m=0}^{M} q_i(k-m)\cdot\{F_{m+1}[\sigma_{ij}(k)] - F_m[\sigma_{ij}(k)]\}$$
$$\text{for } 0 \le i < j \le N$$

$$H^k_{N+\ell,Ni+j-i(i+1)/2} = \sum_{m=0}^{M} q_i(k-m)\cdot\{F_{m+1}[\sigma_{ij}(k)] - F_m[\sigma_{ij}(k)]\}$$
$$\text{for } 0 \le i < \ell < j \le N$$

- $J^{k-1} = [J^{k-1}_{rs}]_{(2N-1)\times N(N+1)/2}$

$$J^k_{j,Ni+j-i(i+1)/2} = \sum_{m=0}^{M} q_i(k-m)\cdot b_{ij}(k), \text{ for } 0 \le i < j \le N$$

$$J^k_{N+\ell,Ni+\ell-i(i+1)/2} = \sum_{m=0}^{M} q_i(k-m)\cdot \sum_{j\le \ell} b_{ij}(k), \text{ for } 0 \le i < \ell < j \le N$$

- $J^k_{rs} = 0$.

For the other entries of matrix J^k

- $$J_k = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_{2N-1} \end{bmatrix} = [H^{k-1}J^{k-1}]_{(2N-1)\times N(N+1)}$$

where each f_i is a row vector of dimension $N(N+1)$

- $$Z'(k) = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_{2N-1} \end{bmatrix}$$
$$= [y_1(k), \ldots, y_N(k), U_1(k) - q_1(k), \ldots, U_{N-1}(k) - q_{N-1}(k)]^T$$

Step 3. Initialize sequential Kalman filtering

- Set $b^0 = b(k-1)$, $\sigma^0 = \sigma(k-1)$
- $P^0 = P_{k-1} + D$, where $D = \begin{bmatrix} D_b & \\ & D_\sigma \end{bmatrix}$, $D_b = \mathrm{diag}[d_b, \ldots, d_b]$ is a covariance matrix of $W(k)$, and D_σ is a constant semipositive matrix.

Step 4. Sequential Kalman filtering iteration

For $i = 1, 2, \ldots, 2N-1$

- $g^i = P^{i-1}f_i^T(P^{i-1}f_i^T + r_i)^{-1}$
- $P^i = P^{i-1} - g^i f_i P^{i-1}$
- $\delta^i = y_i(k) - f_i b(k-1)$
- Truncation

$$\alpha' = \max_{0\le\alpha\le 1}\left[\alpha \,\middle|\, 0 \le \begin{pmatrix} b^{i-1} \\ \sigma^{i-1} \end{pmatrix} + \alpha\delta^i g^i \le 1\right]$$

$$\text{set } \begin{pmatrix} b^i \\ \sigma^i \end{pmatrix} = \begin{pmatrix} b^{i-1} \\ \sigma^{i-1} \end{pmatrix} + \alpha'\delta^i g^i$$

- Normalization

For $m = 1, 2, \ldots, N-2$

$$\beta_m = \sum_{j=m+1}^{N} b^i_{mj}$$

$$b^i_{mj} = b^i_{mj}/\beta_m, \; j = m+1, \ldots, N$$

Step 5. Predict states

- Set $P_k = P^{2N-1}$

$$\begin{bmatrix} b(k) \\ \sigma(k) \end{bmatrix} = \begin{bmatrix} b^{2N-1} \\ \sigma^{2N-1} \end{bmatrix}$$

$k = k+1$, go to Step 1 for the next interval.

Although the extended Kalman filtering algorithm for the O-D parameter estimation cannot guarantee an unbiased minimum variance estimate, it has been found to be very efficient in many applications (*29*). However, such a nonlinear model remains somewhat complex because of the large dimension of its state vector. For online applications, one may choose to trade accuracy, to some extent, for computing efficiency. To properly execute the preceding algorithm, care should be exercised on the following issues (*30*):

- The variance–covariance matrices are not known. As often used in the control study, they can be assumed as diagonal matrices with their diagonal element values lying within interval (0,1). The observation error variance $\{r_i\}$ is often presumed to be the square of an estimated flow measurement error.
- As a general guideline, the segment length should be consistent with two basic requirements: containing no more than one pair of on- and off-ramps for modeling, and computational convenience.
- In terms of the time interval selection, theoretically, a shorter time duration can better accommodate the dynamic O-D flow pattern. However, to be both effective and efficient in applications, the time interval should be shorter than the travel time of each segment and between 1 and 5 minutes, depending on the level of congestion.

NUMERICAL EXAMPLE

This section presents two numerical results with example networks. The first small freeway network is designed to evaluate the proposed model's performance with respect to its initial values and potential measurement errors in travel time. A large freeway network, based on the I-95 freeway corridor, is presented to demonstrate the model's potential for real-world applications. The sensitivity analyses were performed with the following procedures:

Step 0. Generation of data set for experimental analysis. To generate a meaningful data set for numerical analysis, the example freeway system under the presumed time series O-D percentages was simulated with AIMSUN 4.0 (*31*) to produce the time-dependent link traffic volumes. The traffic flow data were collected at an interval of 1 minute over the entire simulation duration. Figure 4 presents the example freeway section for the model sensitivity test.

Step 1. Random generation of several sets of initial values. To test the model performance under a different set of initial values, this study has generated the following five experimental sets for use in executing the proposed solution algorithm:

- Ib1 = (0.20 0.32 0.48 0.20 0.32 0.48), the exact initial value set;
- Ib2 = (0.33 0.33 0.33 0.33 0.33 0.33), the uniform initial value set;
- Ib3 = (0.25 0.29 0.46 0.25 0.29 0.46), the initial value set with certain random variation;
- Ib4 = (0.40 0.30 0.30 0.40 0.30 0.30), the initial value set with random variations more than those in Ib3; and
- Ib5 = (0.50 0.40 0.10 0.50 0.40 0.10), the initial value set with random variations more than those in Ib4.

Step 2. Generation of travel time variation. Aside from the set of the initial values, the actual distribution of travel times is one of the important factors that could influence the estimated O-D proportions. To test the robustness of the proposed travel time formulation, the exact initial value set Ib1 is selected for executing the estimation algorithm, and the average travel time is randomly increased or decreased between 5%, 10%, and 15% from the average travel time for model performance evaluation.

Step 3. Evaluation of performances. The root-mean-square error (RMSE) used as the evaluation criterion is defined as follows:

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{N}}$$

The estimation results for these sets of O-D proportions are presented in Figure 5. The RMSE statistics are reported in Table 1. It is noticeable that, with the reasonable range of the initial values, the estimation results with the proposed algorithms are quite stable and vary in a small range as indicated in Figure 5.

Table 2 presents the RMSE under different levels of travel time variation, and Figure 6 presents the estimated O-D proportions under various travel times. It can be noted that the proposed model yields quite stable results, where the RMSE remains nearly constant even when the average travel times are up to 15% measurement errors.

To demonstrate the potential of the proposed O-D estimation model, this study has evaluated its performance with the I-95 freeway corridor between the I-495 and I-695 beltways in Maryland, which consists of seven main interchanges, 12 on-ramps, and 14 off-ramps. The total number of O-D pairs for this network amounts to 120. For convenience of model formulations, each interchange is represented with only one pair of on-ramp and off-ramp, and the network is thus reduced to seven pairs of on-ramps and off-ramps, and 36 O-D sets as indicated in Figure 7.

Table 3 presents the input O-D demands over 1-hour simulation with a unit interval of 2 minutes. Using the first time-period O-D proportions as the initial set of O-D values, the average RMSE for all O-D pairs is 0.0454. This distribution of RMSE for all estimated O-D pairs is presented in Table 4, and the estimated results for two example O-D pairs are presented in Figures 8 and 9.

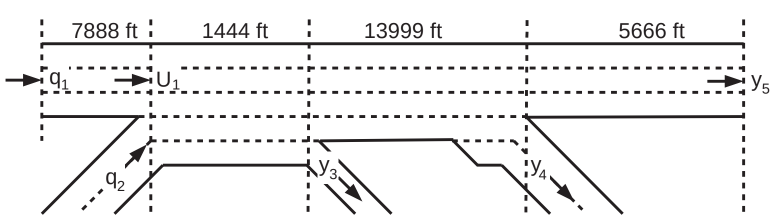

FIGURE 4 Small example freeway section for model sensitivity test.

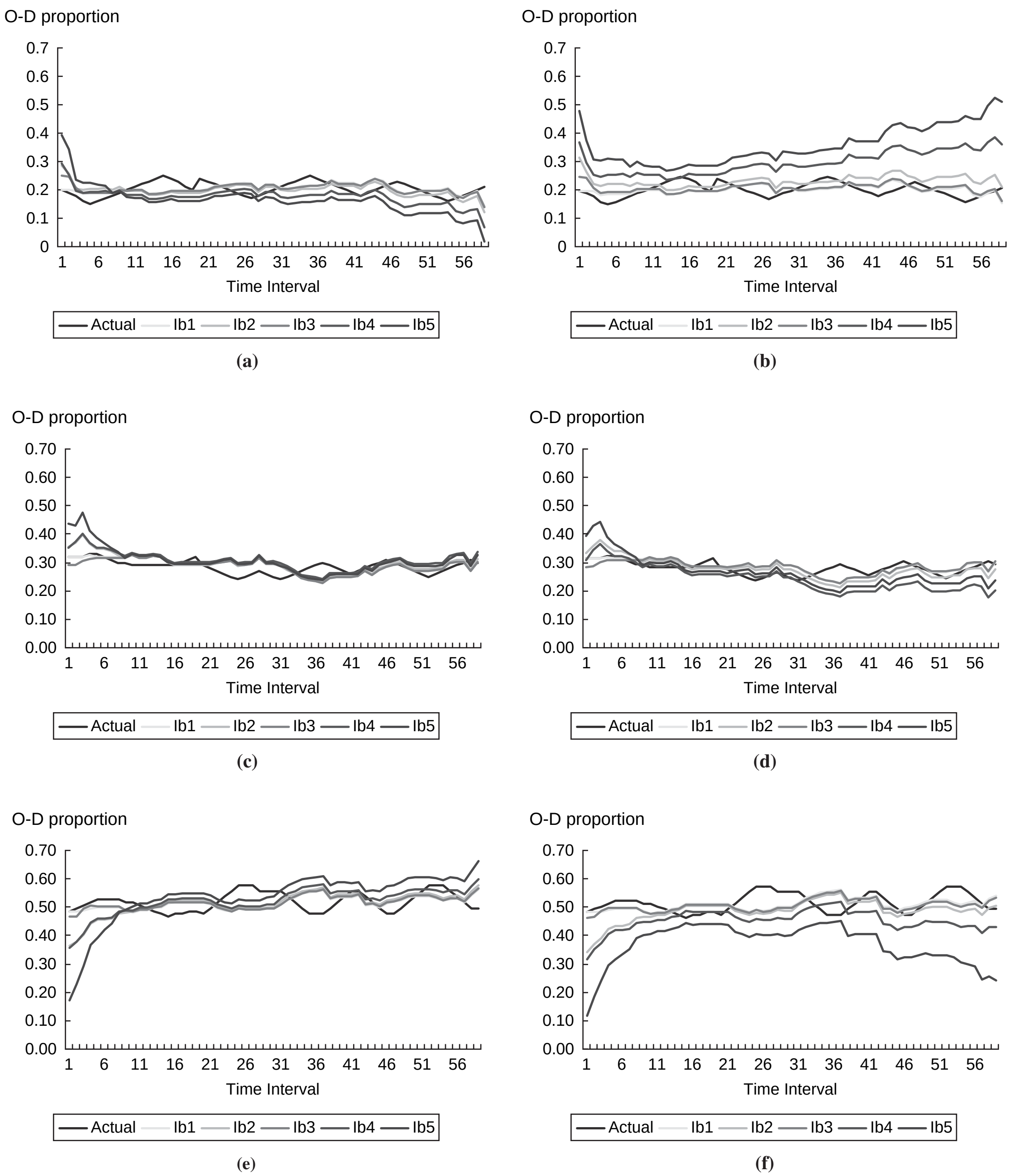

FIGURE 5 Estimation results with different sets of initial values: (*a*) b13, (*b*) b23, (*c*) b14, (*d*) b24, (*e*) b15, and (*f*) b25.

TABLE 1 RMSE Statistics with Different Initial Values

Initial Value	RMSE (b_{13})	RMSE (b_{14})	RMSE (b_{15})	RMSE (b_{23})	RMSE (b_{24})	RMSE (b_{25})	Avg RMSE
Ib1	0.0315	0.0237	0.0301	0.0316	0.0232	0.0308	0.0285
Ib2	0.0316	0.0296	0.0452	0.0424	0.0301	0.0486	0.0379
Ib3	0.0326	0.0232	0.0301	0.0303	0.0223	0.0296	0.0280
Ib4	0.0469	0.0291	0.0389	0.0445	0.0269	0.0401	0.0377
Ib5	0.0570	0.0270	0.0506	0.0741	0.0233	0.0737	0.0510

TABLE 2 RMSE Statistics Under Different Levels of Travel Time Variation

Travel Time Variation	RMSE (b_{13})	RMSE (b_{14})	RMSE (b_{15})	RMSE (b_{23})	RMSE (b_{24})	RMSE (b_{25})	Avg RMSE
0%	0.0315	0.0237	0.0301	0.0316	0.0232	0.0308	0.0285
5%	0.0288	0.0275	0.0424	0.0297	0.0275	0.0437	0.0333
10%	0.0327	0.0210	0.0333	0.0394	0.0192	0.0375	0.0305
15%	0.0317	0.0240	0.0330	0.0387	0.0283	0.0349	0.0318

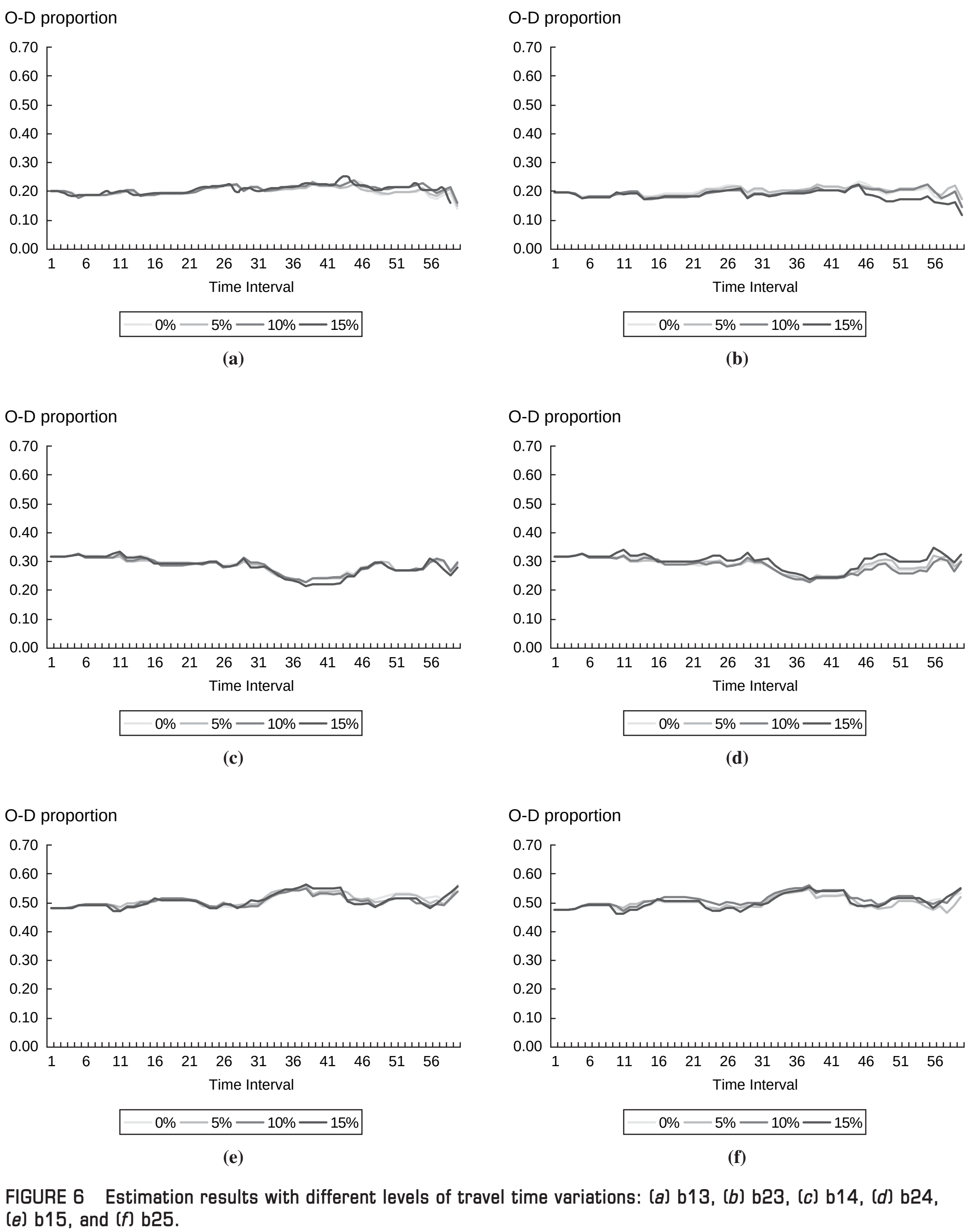

FIGURE 6 Estimation results with different levels of travel time variations: (*a*) b13, (*b*) b23, (*c*) b14, (*d*) b24, (*e*) b15, and (*f*) b25.

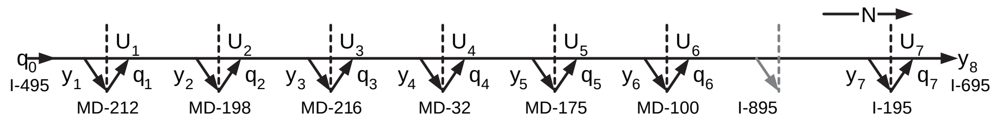

FIGURE 7 Graphic illustration of main interchanges for I-95 freeway corridor.

In Figure 8, the pattern of the estimated O-D proportions is very close to the real pattern and the RMSE is 0.0185. In Figure 9, the variation of the beginning time interval appears flatter than the real pattern. This might be due to the setting of the covariance matrix and the relatively smaller O-D demand (206 vehicles per hour compared with 827 vehicles per hour for b04 as indicated in Table 3). However, the estimation result is still within a reasonable range (the RMSE is 0.0295) and the pattern of the latter time interval is getting closer to the real pattern after several updates of the covariance matrix.

CONCLUSIONS

This study has presented a new dynamic model for estimating the time-varying freeway O-D matrices. The proposed model features its robustness in minimizing the impacts of travel time variability on the estimation results. With the embedded travel time function, the proposed model can reliably estimate the dynamic O-D pairs that may be distributed over a relatively long distance and take a relatively long travel time. The reduced number of parameters also enables the proposed model to have better potential for efficient applications. To ensure the applicability of the proposed model for a large-scale network, the study has constructed a simulator for the I-95 freeway corridor in Maryland with the simulation program AIMSUN 4.0 and performed the model applicability evaluation. The results indicate that the proposed model can yield reasonable estimates of dynamic O-D proportions for large freeway corridors.

One of the critical issues that remains to be investigated in the development of a dynamic O-D model is how best to approximate the initial values of each O-D set from measurable information so that the estimation process with the recursive computing algorithm such as extended Kalman filtering can evolve efficiently to a reliable and stable state.

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TABLE 3 Input O-D Demand for 1-h Simulation

Origin \ Destination	0	1	2	3	4	5	6	7	8	*Oi*
0	—	1,371	1,190	749	827	692	560	570	375	6,334
1	—	—	212	303	186	214	226	264	210	1,615
2	—	—	—	484	395	196	348	224	311	1,958
3	—	—	—	—	344	263	170	218	390	1,385
4	—	—	—	—	—	166	206	306	980	1,658
5	—	—	—	—	—	—	185	134	248	567
6	—	—	—	—	—	—	—	318	1,031	1,349
7	—	—	—	—	—	—	—	—	1,338	1,338
8	—	—	—	—	—	—	—	—	—	—
Dj	—	1,371	1,402	1,536	1,752	1,531	1,695	2,034	4,883	16,204

TABLE 4 RMSE Statistics for I-95 Freeway Corridor

O-D pair	b_{01}	b_{02}	b_{03}	b_{04}	b_{05}	b_{06}	b_{07}	b_{08}	b_{12}	b_{13}	b_{14}	b_{15}
RMSE	0.0396	0.0451	0.0197	0.0185	0.0186	0.0231	0.0245	0.0472	0.0361	0.0525	0.0401	0.0451
O-D pair	b_{16}	b_{17}	b_{18}	b_{23}	b_{24}	b_{25}	b_{26}	b_{27}	b_{28}	b_{34}	b_{35}	b_{36}
RMSE	0.0411	0.0405	0.0382	0.0512	0.0393	0.0297	0.0373	0.0424	0.0436	0.0349	0.0543	0.0462
O-D pair	b_{37}	b_{38}	b_{45}	b_{46}	b_{47}	b_{48}	b_{56}	b_{57}	b_{58}	b_{67}	b_{68}	avg.
RMSE	0.0450	0.0438	0.0426	0.0295	0.0622	0.0946	0.0815	0.0850	0.1035	0.0688	0.0688	0.0454

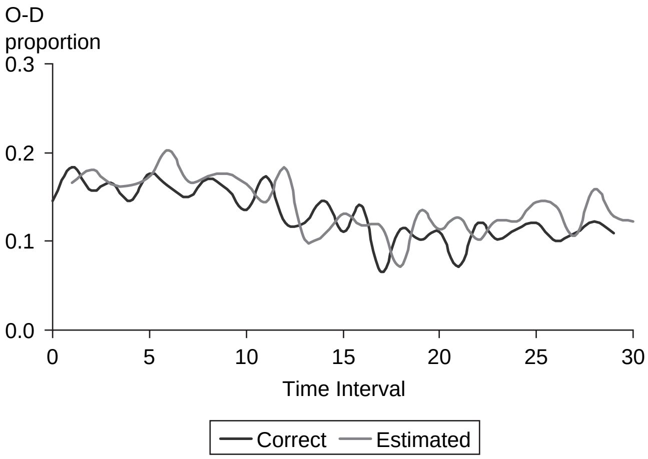

FIGURE 8 Estimation result of O-D pair b04 (with 0.0185 RMSE).

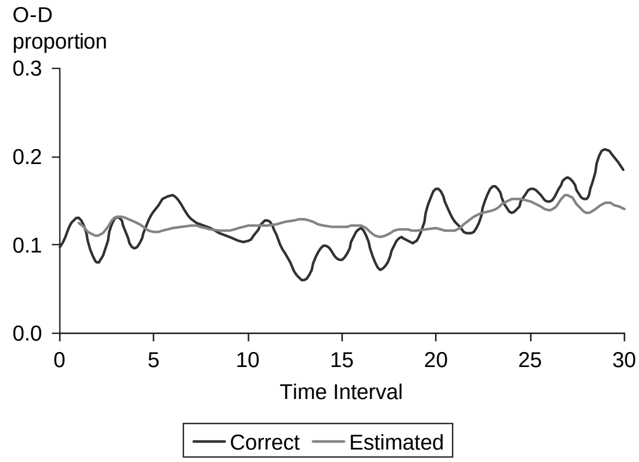

FIGURE 9 Estimation result of O-D pair b46 (with 0.0295 RMSE).

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