AN INTEGRATED BUS-BASED PROGRESSION SYSTEM FOR ARTERIALS HAVING HEAVY TRANSIT FLOWS

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Transit Signal Priority (TSP)

- Transit system
- Active control strategies
- Bus-based progression
- Bus operational features

Source: Sustainable Transportation in the Netherlands
Outline

• Literature Review

• Problem Nature and Modelling Framework

• Methodology

• Case Study

• Conclusions and Future Study
Literature Review

• The concept of TSP has been developed since late 1960s (Smith, 1968).
  - Active strategies detect the arrival of buses and grant a priority to them. (Ludwick & John, 1974; Dion & Hesham, 2005)
  - Passive strategies do not recognize the presence of buses, but predetermine the signal timings to facilitate bus movements. (Urbanik, 1977)

• Limitations of TSP strategies
  - Significant negative impact to cross-street traffic if the target arterials experience heavy bus volumes
  - May interrupt the conventional signal progression design
Signal progression, first presented by Morgan and Little (1964), is studied mainly for passenger cars.

- Allow some vehicles to pass consecutive intersections without encountering red phases.
- Reduce accidents.
- MAXBAND (Little et al., 1981)
- MULTIBAND (Gartner et al., 1990)

Bus progression is a promising passive strategy to improve the operational efficiency of transit systems with
- minimized negative impact to cross-street traffic
- benefits to transit vehicles on an arterial
Outline

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Critical issues

- Dwell time at bus stops
- Dwell time uncertainty
- Bus stop capacity
- Competition between buses and PCs
Critical issues

Dwell time at bus stops

Bus stop capacity

Competition between buses and PCs

Transit vehicles, impacted by the dwell time at stops, may not stay in the green band designed for passenger cars.
Critical issues

Deterministic dwell time VS. Stochastic dwell time

The stochastic nature of bus dwell time should be considered when studying bus progression.
Critical issues

Is the **wider** band always **better**?

The number of buses in a band shall not exceed the capacity of the bus stop to prevent the formation of bus queues.
Critical issues

Dwell time at bus stops

Dwell time uncertainty

Bus stop capacity

Competition between buses and PCs

The bus band and passenger-car band may need to be optimized concurrently.
Critical issues

Modelling Framework

- A deterministic model for dwell time at bus stops
- An evaluation module
- A progression model for buses

Following the concept of MAXBAND, a Mixed-Integer Linear Programming model is developed. Taking advantage of the results produced from the deterministic model, an evaluation module is developed to fully account for the stochastic nature of bus dwell time.

- An enhanced deterministic model
- Integrating bus stop uncertainty
- An enhanced evaluation module
- A integrated model for both buses and PCs
- A progression model for passenger car's benefits
Outline

• Literature Review

• Problem Nature and Modelling Framework
  - A deterministic model
  - An evaluation module
  - An integrated model

• Methodology

• Case Study

• Conclusions and Future Study
Methodology

- Mixed Integer Linear Programming
- Objective function
  \[ \text{Max } \sum_i \varphi_i b_i + \sum_i \bar{\varphi}_i \bar{b}_i \]
- Constraints
  - Interference constraints
    - \( w_i - 0.5 b_i \geq 0 \) \( \forall i \)
    - \( w_i + 0.5 b_i \leq g_i \) \( \forall i \)
    - \( \bar{w}_i - 0.5 \bar{b}_i \geq 0 \) \( \forall i \)
    - \( \bar{w}_i + 0.5 \bar{b}_i \leq g_i \) \( \forall i \)

Discussion of parameter \( \varphi \):
- \( \varphi \) is a weight factor
- The value of \( \varphi \) depends on the number of buses passing intersection \( i \) using the synchronized phase.

Taking each bus stop as a control point
Methodology

- Constraints
  - Progression constraints
    - For links with bus stops
      - Outbound
        \[ \theta_i + w_i + t_i + adt_i = \theta_{i+1} + w_{i+1} + n_{i+1}C \]
      - Inbound
        \[ -\theta_i - \bar{r}_i + \bar{w}_i + \bar{t}_i + adt_i = -\theta_{i+1} - \bar{r}_{i+1} + \bar{w}_{i+1} + \bar{n}_{i+1}C \]
    - For other intersections
      - Outbound
        \[ \theta_i + w_i + t_i = \theta_{i+1} + w_{i+1} + n_{i+1}C \]
      - Inbound
        \[ -\theta_i - \bar{r}_i + \bar{w}_i + \bar{t}_i = -\theta_{i+1} - \bar{r}_{i+1} + \bar{w}_{i+1} + \bar{n}_{i+1}C \]
Methodology

A deterministic model

Constraints (bus stop capacity)

- Bandwidth limit
  - Outbound
    - $w_i - 0.5 \times b_i^{\text{max}} \leq M \times x_i$
    - $w_i + 0.5 \times b_i^{\text{max}} \geq g_i - M \times (1 - x_i)$

- Inbound
  - $\bar{w}_{i+1} - 0.5 \times b_i^{\text{max}} \leq M \times \bar{x}_{i+1}$
  - $\bar{w}_{i+1} + 0.5 \times b_i^{\text{max}} \geq g_{i+1} - M \times (1 - \bar{x}_{i+1})$

- These constraints are only for upstream intersections of bus stops

The center of a band should be either close to the start of green ($x_i = 0$) or close to the end of green ($x_i = 1$) to make sure that the potential band is realistic.
Methodology

Other Constraints

• Bandwidth equality

• For links without bus stops
  \[ b_{i+1} \geq a \times b_i + \beta \times \sigma_i \]

• Dwell time uncertainty

• For links with bus stops
  \[ \overline{b}_i \geq a \times \overline{b}_{i+1} + \beta \times \overline{\sigma}_{i+1} \]

A deterministic model

For those buses passing the upstream intersection during \( a \downarrow b_i \), if the dwell time uncertainty is within a specific range, the departing band should accommodate them.

\( a \): between 0 and 1
\( \beta \): indicating the tolerance of dwell time uncertainty
## Methodology

- **Objective Function**
  \[
  \text{Max } \sum_i \varphi_i b_i + \sum_i \varphi_i \bar{b}_i
  \]

- **Constraints**
  \[
  w_i - 0.5b_i \geq 0, \quad w_i + 0.5b_i \leq g_i, \quad w_i - 0.5\bar{b}_i \geq 0, \quad w_i + 0.5\bar{b}_i \leq g_i \quad \forall i
  \]

### Interference constraints

For adjacent intersections between which a stop is located

\[
\begin{align*}
\theta_k + w_k + t_k + \text{ad}t_k &= \theta_{k+1} + w_{k+1} + n_{k+1}C \\
-\theta_k + \bar{r}_k + \bar{w}_k + \bar{t}_k + \text{ad}t_k &= -\theta_{k+1} + \bar{r}_{k+1} + \bar{w}_{k+1} + \bar{n}_{k+1}C
\end{align*}
\]

\[
\begin{align*}
w_k - 0.5 \times b_k^{\text{max}} &\leq M \times x_k \\
w_k + 0.5 \times b_k^{\text{max}} &\geq g_k - M \times (1 - x_k) \\
\bar{w}_{k+1} - 0.5 \times b_{k+1}^{\text{max}} &\leq M \times x_{k+1} \\
\bar{w}_{k+1} + 0.5 \times b_{k+1}^{\text{max}} &\geq g_{k+1} - M \times (1 - x_{k+1})
\end{align*}
\]

\[
\begin{align*}
b_{k+1} &\geq a \times b_k + \beta \times \sigma_k \\
\bar{b}_k &\geq a \times \bar{b}_{k+1} + \beta \times \bar{\sigma}_{k+1}
\end{align*}
\]

### Progression constraints

\[
\begin{align*}
\theta_k + w_k + t_k &= \theta_{k+1} + w_{k+1} + n_{k+1}C \\
-\theta_k + \bar{r}_k + \bar{w}_k + \bar{t}_k &= -\theta_{k+1} + \bar{r}_{k+1} + \bar{w}_{k+1} + \bar{n}_{k+1}C
\end{align*}
\]

\[
\begin{align*}
b_k &= b_{k+1} \\
\bar{b}_k &= \bar{b}_{k+1}
\end{align*}
\]

### Bandwidth constraints

\[
\begin{align*}

\end{align*}
\]

### Dwell time uncertainty

\[
\begin{align*}

\end{align*}
\]
Methodology

- A deterministic model
- An evaluation module
- A progression model for buses
- Integrating passenger car’s benefits
- An enhanced deterministic model
- An enhanced evaluation module
- A integrated model for both buses and PC s
By adjusting parameters in this critical constraint, one may have multiple sub-optimal solutions. They will be evaluated and ranked, fully taking the stochastic nature of bus dwell time into consideration.
Methodology

\[ b_{i+1} \geq a' \times b_i \]

- Computational complexity
- Still describing the relation between the arriving bandwidth and the departing bandwidth
- Still ensuring a relatively large departing bandwidth based on its arriving bandwidth
- Although the dwell time variance is no longer considered in the constraints, the analysis to sub-optimal solutions applies a more rigorous method to assess the impact of dwell time variance.

Discussion of parameter \( a' \):
- Different values lead to different “optimal” solutions
  - A Greater value for \( a' \) ensures a higher probability of a bus to keep in the band
  - A Smaller value for \( a' \) allows a larger arriving bandwidth
- A too Large value for \( a' \) leads to meaningless upstream bands.
How effective a signal plan is highly depends on the relation between each pair of bands arriving to and departing from a bus stop.

To evaluate the sub-optimal solutions, this module computes the expectation of the fraction of the arriving bandwidth which can be effectively utilized.

This expectation is called “effective bandwidth”.

\[ b_{i+1} \geq a \times b_i \]
Methodology

• In order to compute the effective bandwidths, one first needs to calculate the probability of a bus to keep in the band. 

$$P(x) = \Phi\left(\frac{0.5b_{i+1} - x_i}{\sigma_i}\right) - \Phi\left(\frac{-0.5b_{i+1} - x_i}{\sigma_i}\right)$$

$$N(x_{i+1}, \sigma_i) = N(x_i, \sigma_i)$$

$$\mu = x_i, \quad \sigma = \sigma_i$$

$$x_i$$

Distance

Time
Methodology

- Probability for the bus coming at time $x$ to stay in the downstream band
  - For outbound
    \[ P(x) = \Phi\left(\frac{0.5b_{i+1} - x}{\sigma_i}\right) - \Phi\left(\frac{-0.5b_{i+1} - x}{\sigma_i}\right) \]
  - For inbound
    \[ \overline{P(x)} = \Phi\left(\frac{0.5\overline{b}_k - x}{\sigma_k}\right) - \Phi\left(\frac{-0.5\overline{b}_k - x}{\sigma_k}\right) \]

- Calculate the “effective bandwidth”
  - $P(x)dx$ is the “effective” part among a short period of time $dx$
  - The “effective bandwidth” can be calculated by
    - For outbound
      \[ b^e_i = \int_{-0.5b_i}^{0.5b_{i+1}} P_i(x)dx \]
    - For inbound
      \[ \overline{b}^e_{i+1} = \int_{-0.5b_{i+1}}^{0.5b_{i+2}} \overline{P}_{i+1}(x)dx \]

Dwell time uncertainty

For an intersection that is not at upstream of a bus stop:

\[ b^e_i = b_i, \overline{b}^e_{i+1} = \overline{b}_{i+1} \]
Methodology

• A larger “effective bandwidth” indicates a higher fraction of the buses which can stay in both the arriving and departing bands.

• Each solution from the deterministic model will generate $2m$ effective bandwidths, an outbound one and an inbound one for each intersection, where $m$ is the number of intersections.

• The solution giving the maximum sum of effective bandwidths can be considered as the optimal solution for the model.
Methodology

Cycle length, green split, travel time, estimated bus dwell time....

\[ b_{i+1} \geq a' \times b_i \]

Each solution has a set of bandwidths and offsets

Find the solution with the maximum total effective bandwidths
Methodology

- A deterministic model
- An evaluation module
- A progression model for buses

Integrating passenger car’s benefits

- An enhanced deterministic model
- An enhanced evaluation module
- A integrated model for both buses and PC s
Designing bus bands causes potential interruption for passenger car movements.

Even with the same bus bands, the benefit for passenger cars can be different among signal plans.
Therefore, the bus bands and the passenger car bands need to be optimized concurrently.

- Revising the objective function to include bands for both types of vehicles.
- Revising the constraints to express passenger car bands.

- When comparing the sub-optimal solutions, both effective bandwidths for buses and passenger car bandwidths are considered.
Methodology

- **Objective function**
  \[ \text{Max } k(\sum_{i} q_i b_i + \sum_{i} \bar{q}_i \bar{b}_i) + n(b^c + \bar{b}^c) \]

- **Additional constraints**
  - An enhanced deterministic model
  \[ (1-k)(\sum_{i} q_i b_i + \sum_{i} \bar{q}_i \bar{b}_i) \geq k(1-k) n(b^c + \bar{b}^c) \]
  - Constraints to express passenger-car bands
    \[ w_i^c - 0.5b^c \geq 0 \quad w_i^c + 0.5b^c \leq g_i \quad \bar{w}_i^c - 0.5\bar{b}^c \geq 0 \quad \bar{w}_i^c + 0.5\bar{b}^c \leq g_i \]
    \[ \theta_i + w_i^c + t_i^c + n_i^C = \theta_{i+1} + w_{i+1}^c + n_{i+1}^C \]
    \[ -\theta_i - r_i + \bar{w}_i^c + \bar{t}_i^c + \bar{n}_i^C = -\theta_{i+1} - r_{i+1} + \bar{w}_{i+1}^c + \bar{n}_{i+1}^C \]

- **Ratio between numbers of passengers on two types of vehicles**

- **Competition between buses and PCs**
  - \( k < 1 \): Passengers on PCs are less
  - \( k > 1 \): Passengers on buses are less
Methodology

- Enhancement to the stochastic analysis
  - The ranking index of a sub-optimal solution includes both effective bandwidth of bus bands and passenger-car bands

\[ R = k \left( \sum_{i=1}^{n-1} b_i^e + \sum_{i=2}^{n} \overline{b}_i^e \right) + (n - 1) \left( b^c + \overline{b}^c \right) \]

- Total effective bandwidths
- Total passenger-car bandwidths

An enhanced evaluation module
Methodology

Cycle length, green split, travel time, estimated bus dwell time....

\[ b_{i+1} \geq a' \times b_i \]

Each solution has a set of bandwidths and offsets

Find the solution with the maximum ranking index
Methodology

A deterministic model

An evaluation module

A progression model for buses

Dwell time at bus stops

Following the concept of MAXBAND, a Mixed-Integer Linear Programming model is developed.

By adjusting a parameter in the MILP, multiple sub-optimal solutions will be produced and evaluated.

Accounting for the stochastic nature of bus dwell time.

An enhanced evaluation module

Bus stop capacity

An integrated model for passenger cars

Dwell time uncertainty

A progression model for buses

Competition between buses and PCs

Benefits of both buses and passenger cars are considered.
Outline

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Case Study

• Case Design

- Cycle length is 150 seconds;
- Green times at intersections are 99, 77, 66, 75, and 60 seconds, respectively;
- The dwell time: bus stop 1: N(30,9); bus stop 2: N(27,7); bus stop 3: (24,9);
- The bus stop capacity is 2 buses at each direction, and the confidence parameter \( p \) equals 0.95; then the maximal bus bandwidth could be computed as 50 seconds
- For each direction along the arterial, the bus volume is 60 veh/h, with an average headway of 1.0 minute and the passenger car volume is 750 veh/h;

<table>
<thead>
<tr>
<th>Link</th>
<th>Link length (ft)</th>
<th>travel time</th>
<th>With bus stop?</th>
</tr>
</thead>
<tbody>
<tr>
<td>I ↔ II</td>
<td>906</td>
<td>20</td>
<td>Yes</td>
</tr>
<tr>
<td>II ↔ III</td>
<td>948</td>
<td>21</td>
<td>No</td>
</tr>
<tr>
<td>III ↔ IV</td>
<td>1250</td>
<td>28</td>
<td>Yes</td>
</tr>
<tr>
<td>IV ↔ V</td>
<td>725</td>
<td>16</td>
<td>Yes</td>
</tr>
</tbody>
</table>

With bus stop?
Case Study

• Models to be evaluated
  • Model-1: MAXBAND with fixed phase sequences
  • Model-2: A direct extension of MAXBAND by adding the average bus dwell time to the travel time on the links having a bus stop.
  • Model-3: The proposed deterministic model
  • Model-4: The proposed deterministic model with the evaluation module
  • Model-5: The proposed integrated model

• The MILP is solved with LINGO. The evaluation module is conducted with R studio.
Case Study

- **Task 1:** Bandwidths and performance measures generated by bus progression models will first be compared to verify the necessity of the evaluation module.

- **Task 2:** Then the signal plans generated by all Models will be applied in the simulation software, VISSIM, and will be evaluated based on the average delays and number of stops.

- **Task 3:** Sensitivity Analysis will then be conducted with respect to the number of passengers on buses to assess the stability of the proposed integrated model.
Case Study

MAXBAND with extension

The deterministic model

The deterministic model + the evaluation stage

Fixed bandwidths

Varying bandwidths

Bandwidths limited by the capacity constraints
Case Study

To verify the necessity of the evaluation module, several sets of parameters for Model-3 are tested.

\[ b_{i+1} \geq a \times b_i + \beta \times \sigma_i \]

<table>
<thead>
<tr>
<th>Model</th>
<th>α</th>
<th>β</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model-3-1</td>
<td>0.3</td>
<td>1</td>
<td>0</td>
<td>102</td>
<td>101</td>
<td>40</td>
<td>38</td>
</tr>
<tr>
<td>Model-3-2</td>
<td>0.3</td>
<td>2</td>
<td>0</td>
<td>107</td>
<td>104</td>
<td>38</td>
<td>43</td>
</tr>
<tr>
<td>Model-3-3</td>
<td>0.5</td>
<td>1</td>
<td>0</td>
<td>105</td>
<td>98</td>
<td>37</td>
<td>41</td>
</tr>
<tr>
<td>Model-3-4</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>99</td>
<td>104</td>
<td>25</td>
<td>35</td>
</tr>
<tr>
<td>Model-3-5</td>
<td>0.1</td>
<td>2</td>
<td>0</td>
<td>104</td>
<td>99</td>
<td>41</td>
<td>40</td>
</tr>
</tbody>
</table>

The deterministic model + the evaluation stage
Case study

The models which take bus progression into consideration, are able to offer operational benefits to bus vehicles on the target arterial, evidenced by reduction in the average bus delay.

Model-1: MAXBAND with fixed phase sequences
Model-2: A direct extension of MAXBAND
Model-4: The proposed deterministic model with the evaluation ranking stage
Model-5: The proposed integrated model

Model-2 and 4 outperform Model-1, and Model-5 outperforms both Model-2 and Model-4
Case Study

• It can be expected that the integrated model should be only applied when the difference between numbers passengers on two types of vehicles is small.

• When the number of passengers on buses dominates that on passenger cars, bus progression model may be preferred, and vice versa.

• The system performance is quite sensitivity to the preference factor $k$

$$\text{Max } k \left( \sum_i q_i b_i + \sum_i \bar{q}_i \bar{b}_i \right) + n \left( b^c + \bar{b}^c \right)$$

<table>
<thead>
<tr>
<th>Loading factor on buses</th>
<th>Passenger ratio k</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>0.8</td>
</tr>
<tr>
<td>18</td>
<td>1.2</td>
</tr>
<tr>
<td>30</td>
<td>2</td>
</tr>
<tr>
<td>7.5</td>
<td>0.5</td>
</tr>
</tbody>
</table>
Case Study

- Model-5, an integrated progression model that accounts for both buses and passenger cars, performs better when the ratio between passengers on the two types of vehicles is close to 1, as expected.

- This may be because the constraints in the integrated guarantee both bus bands and passenger car bands which is unnecessary and may limit the bandwidth for the type of vehicle with significantly higher volume.
Outline

• Literature Review

• Problem Nature and Modelling Framework

• Methodology

• Case Study

• Conclusions and Future Study
Conclusions

• Due to the limited functions of the active transit signal priority control and the strengths of arterial signal progression, this study has developed a bus progression system to facilitate bus movements on an arterial.

• The key features of the developed model include:
  1) the impact of bus dwell time at a bus stop between intersections on the progression design;
  2) the stochastic nature of bus dwell time;
  3) the capacity of bus stops; and
  4) the competition on the green band between buses and passenger cars

• The simulation results demonstrate that the proposed model can reduce both bus passenger delays and average person delays for vehicles in the entire network, compared to the conventional progression models.
Future Research

• Developing a set of rigorous criteria that can compute the trade-off between bus based and passenger-car-based progression models and select the proper one in real time based on the detected traffic conditions

• An extensive sensitivity analysis with field data and simulation experiments
Thank you

• Questions and Comments
Methodology

• How to determine $b_{\text{max}}$?

• Probability of $k$ buses being in a band $i$ is

\[
f(k) = (\lambda b_{i})^{k} \times e^{-\lambda b_{i}} / k!
\]

  • Where, $\lambda$ is bus arrival rate and $b_{i}$ is bandwidth of band $i$

• The probability that the number of buses in a band does not exceed the capacity should be greater than a predetermined $\alpha$, which can be expressed as,

\[
\sum_{k=0}^{C_{\text{ls}}} f(k) \geq \alpha
\]

  • Where, $C_{\text{ls}}$ is the bus stop capacity

• Then $b_{\text{max}}$ can be determined by

\[
\sum_{k=0}^{C_{\text{ls}}} (\lambda b_{i})^{k} \times e^{-\lambda b_{i}} / k! \geq \alpha
\]
Methodology

- How to determine $a$?
- 1) set a minimum band $b_{\downarrow min}$ and $b_{\downarrow max}, k$
- 2) $a^u = \min_k \left( b_{\downarrow min} \right)$ upper bound
- 3) $a^l = \min_k \left( b_{\downarrow max} \right)$ lower bound
- 4) $a^{\min} = \max_k \left( b_{\downarrow max, k} \right)$ Minimum interval
- 5) the number of different values of $a$ is $a^{\uparrow u} - a^{\uparrow l} / a^{\uparrow min}$

- Based on the bandwidth resolution of 1 second

- The smaller one among $b_{\downarrow max}$ and the green time

- A band smaller than that is meaningless operationally.

An evaluation module