Abstract: This paper presents a mixed-integer model for design of arterial signal control strategies during emergency evacuation. The proposed model can effectively take into account various complex operational issues such as critical intersection selection, demand rerouting, and signal timing. The control objective is to maximize the efficiency of the primary evacuation arterial, but not incur excessive waiting time and queues on its side streets. To evaluate the effectiveness of the proposed model under various demand levels and control objectives, this study has employed one major evacuation corridor in Washington D.C. as the target route for numerical experiments. The results of extensive simulation experiments reveal that the proposed method for signal optimization offers the potential for real-world applications. 

Keywords: evacuation, signal control, integer programming, critical intersection, routing

1. INTRODUCTION

During major emergencies and disasters, how to efficiently evacuate the target population has long been a challenge to transportation professionals. A large body of literature has indicated that signal control is one of the critical factors that may significantly impact the efficiency of evacuation operations (Sisiopiku et al., 2004; DTOEM, 2005). However, due to the complex interrelations between evacuee responses and the control objectives, how to effectively design a signal control strategy that can minimize the overall cost of the entire evacuation remains an on-going research issue in the transportation community.

This study is focused mainly on maximizing the evacuation efficiency at the arterial level. It involves the selection of critical intersections on the arterial for executing signal control and the design of cycle length as well as signal settings. The core concept intends to reduce the disturbance of side street traffic to the arterial flow progression, and allow turning movements be taken place only at those selected critical intersections. With an effective signal control system, the main evacuation arterial should be capable of progressively moving its assigned traffic flows, but not over delay those waiting at minor streets for joining the evacuation flow.

The rest of the paper is organized as follows. Next section lists the assumptions and notations of the proposed model. Section 3 gives an in-depth discussion of the model formulation, including both the selection of control objectives and the identification of various operational constraints. Numerical experiments and evaluation results are elaborated in Section 4, with one primary evacuation corridor in Washington D.C. as the target route. Section 5 concludes the paper with research findings.
2. MODEL ASSUMPTIONS AND NOTATIONS

This paper presents an effective model for optimizing arterial signal controls during emergency evacuation. The proposed formulations are based on the following assumptions:
- The time-varying evacuation demand to the arterial has been determined by a higher level evacuation model or other sources;
- Network features are available, including the length, speed and number of lanes on each arterial link as well as the number of approaches on each side street;
- The set of intersections that can receive side street traffic detoured from each non-critical intersection has also been pre-determined.

To facilitate the model presentation, the notations used hereafter are summarized below:

**Parameters**

\( T \): Time horizon of the study;
\( \Delta t \): Update interval of system status;
\( t, t' = 1, ..., T \): Index of time intervals;
\( r, r' = 1, ..., R \): Index of origins;
\( s \): The evacuation destination;
\( i, i' = 1, ..., I \): Index of arterial links;
\( m, m' = 1, ..., M \): Index of intersections;
\( S_T, m = 1, ..., M \): Set of side streets at intersection \( m \);
\( w, w' \in S_T \): Index of side streets at intersection \( m \);
\( d_r(t), r = 1, ..., R \): Demand generated at origin \( r \) during interval \( t \);
\( C_{I_i}, r = 1, ..., R \): Set of connected side streets for traffic from origin \( r \) to enter the arterial;
\( A_D, r = 1, ..., R, w \in C_{I_i} \): Delay for travelling from origin \( r \) to side street \( w \);
\( I_{ij}, i = 1, ..., I \): Length of arterial link \( i \), \( l = \) physical length/speed (unit: no. of \( \Delta t \));
\( Q_i : Flow \ capacity \ of \ arterial \ link \ i \), \( Q = \) saturation flow rate \( \times \) no. of lanes \( \times \) \( \Delta t \) (unit: no. of vehicles);
\( N_i, i = 1, ..., I \): Storage capacity of arterial links, \( N = \) jam density \( \times \) no. of lanes \( \times \) physical length;
\( \Gamma^{-1}(i) \): Set of upstream links of arterial link \( i \);
\( \Gamma(i) \): Set of downstream links of arterial link \( i \);
\( u_m, m = 1, ..., M \): Index of the upstream arterial link of intersection \( m \);
\( d_m, m = 1, ..., M \): Index of the downstream arterial link of intersection \( m \);
\( Q_w : Flow \ capacity \ of \ side \ street \ w \), \( Q = \) saturation flow rate \( \times \) no. of lanes \( \times \) \( \Delta t \) (unit: no. of vehicles);
\( CT \): Evacuation clearance time;
\( x_i^t \): No. of vehicles on link \( i \) at the beginning of \( t \);
\( y_{ij}^t \): No. of vehicles travelling from link \( i \) to link \( j \) during interval \( t \);
\( y_{ij}^t \): Binary variable. \( y_{ij}^t = 1 \) if interval \( t \) is arterial green phase at intersection \( m \)

**Decision Variables:**

\( \delta_m, m = 1, ..., M \): Binary variables. \( \delta_m = 1 \) if intersection \( m \) is critical intersection;
\( C \): Cycle length (unit: no. of \( \Delta t \));
\( g_m, m = 1, ..., M \): Arterial green time of intersection \( m \) (unit: no. of \( \Delta t \));
\( \Delta_m, m = 1, ..., M \): Offset of intersection \( m \) (Unit: no. of \( \Delta t \));
\( \theta_r, r = 1, ..., R, w \in C_{I_i} \): Binary variable. \( \theta_r = 1 \) if demand from \( r \) is diverted to side street \( w \).

3. MODEL FORMULATION

**Objective Functions**

Given the time window \( T \) during an emergency evacuation, the primary objective of traffic operators would be to maximize the total throughput, i.e., the total number of evacuees that can get out of the hazardous area via the evacuation arterial. Since this throughput equals to the total number of vehicles entering the target destination, it can be formulated as Equation (1).

\[
\max x_i^{T+1} = \sum_{t=1}^{T} y_{ih}^t 
\]

If the evacuation time window \( T \) is sufficiently long for all evacuees to get out of the hazardous area, control objective shall be set to minimize the evacuation clearance time, and be formulated as follows.

\[
\min \quad CT \\
\text{s.t.} \quad x_i^{CT+1} = \sum_{r=1}^{R} \sum_{t=1}^{T} d_r(t), CT \leq T 
\]

However, as reported in the literature, maximizing throughput on the main evacuation arterial can cause long queue and delay for side street traffic (Chen, 2005), and thus result in evacuees’ inobservance of the intersection control. Due to such a concern, the proposed model consists of a supplemental objective, which is to optimally control the difference in service level among different locations in the evacuation network.

At the most upstream intersection, one can simply compare the average delay on all approaching links. For each of the other critical intersections \( m \), this paper proposes to compare the average delay for side street(s) at intersection \( m \) with the average delay for all traffic from its upstream intersections. This intends to capture the fact that upstream intersections are closer to the incident site and thus evacuees are likely more panic and thus have lower tolerance to a large delay. Based on this concept, one can formulate the supplemental objective as Equation (3).
To efficiently contend with the proposed multiple objectives for optimizing arterial control, this study employs the popular Hierarchical Optimization Method (HOM) that allows users to rank the selected objectives in a descending order of importance. Each objective function is then minimized sequentially subject to a constraint that does not allow the minimum for the new function to exceed a prescribed fraction of the minimum of the previous function (Eschenauer et al., 1986; Homburg, 1998).

**Network Flow Constraints**

To capture the progression and interactions of vehicles over the evacuation route, this paper uses the generalized cell transmission concept (Liu et al., 2005), a set of link-based formulations that move vehicles from upstream to downstream based on predefined rules. For an evacuation corridor, there exist two groups of network flow constraints.

The arterial links that receive vehicles from its upstream links and send vehicles out to its downstream link can be represented with Equations (4)-(6).

\[
x_{ij}^{t+1} = x_{ij}^t + \sum_{k \in \Gamma(i)} y_{ki}^t - y_{ij}^t - \gamma_{ij} \min \{Q_{ij}^t, n_i t_i - x_{ij}^t\} - \sum_{m \in \Gamma(i)} \sum_{d=t_i}^{t-1} y_{ij}^d
\]

(4)

Those side street links typically will receive vehicles from origins, send vehicles out to its downstream arterial link, and store the vehicles that cannot get on the downstream arterial due to signal control or congestion. Such a process can be captured with Equations (7)-(9).

\[
x_{w}^{t+1} = x_{w}^t + \sum_{r \in \text{CL}_r} y_{rw}^{t} - \gamma_{w,r}^t - \Delta_{rw}^t \min \{Q_{rw}^t, n_r t_r - x_{rw}^t\}
\]

(7)

\[
y_{rw}^t = d_r^{-AD_{rw}} \times \theta_{rw}^t \geq AD_{rw}
\]

(8)

\[
y_{w,r}^t = \min \{Q_{w}^t (1 - \gamma_{w}^t), x_{w}^t\}
\]

(9)

**Routing to Critical Intersections**

Since traffic volumes having the access to the evacuation arterial via minor intersections need to be routed to critical intersections, one can model the re-routing process in the design of arterial control with the following expressions.

**Interrelations between Traffic Control Parameters**

Since a non-critical intersection does not provide side street traffic with a protective phase, its green time for the main arterial movement will equal the cycle time and its offset will be zero. In contrast, the arterial green time at a critical intersection shall be between the minimal green time and the cycle length, depending on the estimated entry flow from the side street. Such constraints are shown in Equations (13)-(16), where \( N \) is a very large positive number and \( \epsilon \) is a very small positive number.

\[
g_w > 0, \quad \Delta_m \geq 0, \quad m = 1,\ldots,M
\]

(13)

\[
\Delta_m \leq C \delta_m, \quad m = 1,\ldots,M
\]

(14)

\[
g_m \geq C - N \delta_m, \quad m = 1,\ldots,M
\]

(15)

\[
g_m \geq C - \epsilon \delta_m, \quad m = 1,\ldots,M
\]

(16)

**Signal Status at Intersection \( m \)**

This set of constraints intend to capture the signal status of intersection \( m \) during time interval \( t \), which shall include the following relations corresponding to Equations (17)-(19):

- For non-critical intersections \( \gamma_m^t \) will always equal to 1.
- At a critical intersection, the value of \( \gamma_m^t \) depends on the corridor cycle time as well as the green time and offset of the intersection. Here \( \text{mod}(a,b) \) is a function to return the remainder after dividing \( a \) with \( b \).

\[
\gamma_m^t \geq 1 - \delta_m, \quad m = 1,\ldots,M
\]

(17)

\[
N \gamma_m^t \geq g_m - \text{mod}(t - \Delta_m, C), \quad m = 1,\ldots,M
\]

(18)

\[
\gamma_m^t \times \{g_m - \text{mod}(t - \Delta_m, C)\} \geq 0, \quad m = 1,\ldots,M
\]

(19)
Other Constraints

To provide a realistic range for the optimized solution, the proposed model also includes nonnegative constraints, initial value of link state variables $x_i^0$, and initial value of flows between links $y_{ij}^0$. In most cases, $x_i^0$ and $y_{ij}^0$ are set to zero for all arterial links and side street links, although $x_i^0$ can be other values to represent the background traffic prior to the evacuation.

4. NUMERICAL EXPERIMENTS

Experimental Design

Figure 1 presents a target area for numerical experiments, which covers the Connecticut Avenue in Washington D.C. area. The entire evacuation route starts from the intersection at K Street and ends at the intersection at Chevy Chase Cir. The length of the evacuation corridor is 8km (5 miles), containing a total of 90 origin nodes, 38 signalized intersections, and 24 intersections with stop/yield sign.

Based on the operational concerns, the application of the signal optimization model is subjected to the following constraints:

- Non-signalized intersections cannot be critical intersections.
- Cycle time will be within a range of 60 to 300 seconds
- Evacuation demand can be directed to any of its downstream critical intersections.

Solution Algorithm

Due to the emergency nature, this study employs an efficient Genetic Algorithm-based heuristic to yield the optimal solutions for selection of critical intersections, their cycle length, and signal settings.

Note that Genetic Algorithm is a search technique widely used to find near optimal solutions to a variety of real-world large-scale optimization issues. Inspired by evolutionary biology, Genetic Algorithms are typically implemented as a computer simulation in which a population of abstract representations (chromosomes) of candidate solutions (individuals) to an optimization problem evolves toward better solutions. The evolution starts from a population of completely random individuals and proceeds in iterations (generations). In each generation, the fitness of the whole population is evaluated, while multiple individuals are stochastically selected from the current population based on their fitness and modified with mutation or crossover to form a new population for the next generation. (Goldberg, 1988; Ladd, 1996)

Based on the GA method, this study has developed the solution algorithm with Visual C++ language, and encoded the solutions to the proposed MIP model with binary strings of 0s and 1s to capture the selection of critical intersections, demand routing, cycle time, green time, and offset.

The developed solution algorithm will first maximize the total throughput, and then switch to the minimization of the evacuation clearance time if the maximized throughput is equal to the total evacuation demand. Once the optimized throughput or clearance time is found, users can specify the percentage of loss in these system measurements they could accept in order to minimize the difference of service levels for different locations in the evacuation network. Then, the algorithm will proceed to minimize the third objective function with this additional constraint on the system measurement.

Experimental Results

To show the effectiveness of the proposed model in design of arterial control strategies during emergency evacuations, this paper employs a pre-calibrated CORSIM simulator to compare the control strategies
generated from the model with two traffic signal plans widely used in evacuation.
- Yellow Flash Plan: Signalized intersections will give arterial traffic yellow flash phase and give side streets traffic red flash phase.
- Minimal Green Plan: Signalized intersections will have a cycle time of 300 seconds while side street traffic only get the minimal green time of 10 seconds.

The evaluation results with simulation experiments are organized as follows, where all the indices for comparison are directly extracted from the CORSIM simulation output files:
- Comparing the throughput and/or evacuation clearance time of Yellow Flash Plan and Minimal Green Plan with those of the optimized control strategy that does not restrict the difference in service level (i.e. have no delay balance considerations) under each demand scenario;
- Presenting throughput and/or evacuation clearance time of the optimized control strategy that intends to restrict the difference in service level for balanced delay under each demand scenario;
- Comparing the average delay and maximal delay of Yellow Flash Plan and Minimal Green Plan with those of the two different optimized control strategies.

Table 1 and Table 2 show the comparison results under Demand Scenarios I and II, whereas the optimized control strategy does not include delay balance consideration. These tables have clearly indicated that the optimized arterial control plans outperform those two widely used signal plans in both demand scenarios.

**Table 1. Throughput Comparison under Demand Scenario I - Without Delay Balance Consideration**

<table>
<thead>
<tr>
<th>Throughput (no. of Vehicles)</th>
<th>Throughput (no. of Vehicles)</th>
<th>Throughput (no. of Vehicles)</th>
<th>Throughput (no. of Vehicles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yellow Flash</td>
<td>Min-Green</td>
<td>Optimized</td>
<td>Optimized</td>
</tr>
<tr>
<td>0.5hr</td>
<td>2629</td>
<td>2827</td>
<td>2574</td>
</tr>
<tr>
<td>1hr</td>
<td>5284</td>
<td>5749</td>
<td>5653</td>
</tr>
<tr>
<td>1.5hr</td>
<td>7611</td>
<td>8507</td>
<td>8745</td>
</tr>
<tr>
<td>2Hr</td>
<td>9102</td>
<td>9494</td>
<td>9624</td>
</tr>
</tbody>
</table>

**Table 2. Throughput Comparison under Demand Scenario II - Without Delay Balance Consideration**

<table>
<thead>
<tr>
<th>Throughput (no. of Vehicles)</th>
<th>Throughput (no. of Vehicles)</th>
<th>Throughput (no. of Vehicles)</th>
<th>Throughput (no. of Vehicles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yellow Flash</td>
<td>Min-Green</td>
<td>Optimized</td>
<td>Optimized</td>
</tr>
<tr>
<td>0.5hr</td>
<td>2629</td>
<td>2827</td>
<td>2805</td>
</tr>
<tr>
<td>1hr</td>
<td>5284</td>
<td>5749</td>
<td>5653</td>
</tr>
<tr>
<td>1.5hr</td>
<td>7598</td>
<td>8505</td>
<td>8595</td>
</tr>
<tr>
<td>2Hr</td>
<td>8880</td>
<td>8880</td>
<td>8880</td>
</tr>
</tbody>
</table>

Comparing Table 3 with Table 1 and Table 2, one can identify that the optimized plan with delay balance consideration may lead to lower throughput or longer clearance time. However, the power of these optimized plans is clearly indicated with Table 4 and Table 5. Table 4 presents the averaged delay of the four control plans under Demand Scenario II, whereas Table 5 presents the maximal delay among all side streets at critical intersections for those two optimized control plans with or without delay balance consideration under both demand scenarios.

**Table 3. Simulated Throughput of the Optimized Plan with Delay Balance Consideration**

<table>
<thead>
<tr>
<th>Demand Scenario</th>
<th>Throughput (no. of Vehicles)</th>
<th>Throughput (no. of Vehicles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario I</td>
<td>0.5hr</td>
<td>2500</td>
</tr>
<tr>
<td></td>
<td>1hr</td>
<td>5363</td>
</tr>
<tr>
<td></td>
<td>1.5hr</td>
<td>8077</td>
</tr>
<tr>
<td></td>
<td>2Hr</td>
<td>9378</td>
</tr>
<tr>
<td>Scenario II</td>
<td>0.5hr</td>
<td>2731</td>
</tr>
<tr>
<td></td>
<td>1hr</td>
<td>5617</td>
</tr>
<tr>
<td></td>
<td>1.5hr</td>
<td>8292</td>
</tr>
<tr>
<td></td>
<td>2Hr</td>
<td>8880</td>
</tr>
</tbody>
</table>

**Table 4. Average Delay under Demand Scenario II**

<table>
<thead>
<tr>
<th>Control Plan</th>
<th>Average Delay (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yellow Flash</td>
<td>20.8</td>
</tr>
<tr>
<td>Min-Green</td>
<td>19.0</td>
</tr>
<tr>
<td>Optimized without</td>
<td>17.0</td>
</tr>
<tr>
<td>Balance Consideration</td>
<td>14.2</td>
</tr>
</tbody>
</table>

**Table 5. Maximal Delay at side streets (Unit: min)**

<table>
<thead>
<tr>
<th>Control Plan</th>
<th>Demand Scenario I</th>
<th>Demand Scenario II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimized without</td>
<td>61.6</td>
<td>40.9</td>
</tr>
<tr>
<td>Balance Consideration</td>
<td>49.2</td>
<td>36.0</td>
</tr>
</tbody>
</table>

Table 4 and Table 5 have indicated that the optimized control plans with delay balance consideration did help the side street traffic but at the price of reduced system throughput or increased evacuation clearance time. Also, the authors have noted that this balance consideration might increase the delay at some side streets, which justifies further efforts to improve the supplemental objective with other operational considerations.

5. CONCLUSIONS

This paper has proposed a mixed-integer programming model for design of arterial signal control strategies during emergency evacuations. The model features its consideration of various
operational constraints and requirements, including the selection of critical intersections, signal settings at critical intersections, and demand rerouting from each origin to its connected critical intersections. Another feature of the proposed approach is its multi-objective structure. It will automatically switch between the two system performance measurements based on the input demand scenario, and try to balance the network delay specified by the users. The numerical experiments with the evacuation route of the Connecticut Avenue in Washington, D.C have indicated the potential of the proposed model compared with two widely used evacuation traffic signal plans under simulated environments.

This study is the authors' initial efforts to design arterial signal control strategies under emergency situations. Future work will be to include other considerations to prevent long queue and waiting time on side-streets, to simultaneously optimize staged evacuation strategies and etc.

REFERENCES
